

Facoltà di Ingegneria
Università degli studi di Pavia

Corso di Laurea Triennale in
Ingegneria Elettronica e Informatica

Campi Elettromagnetici e Circuiti I

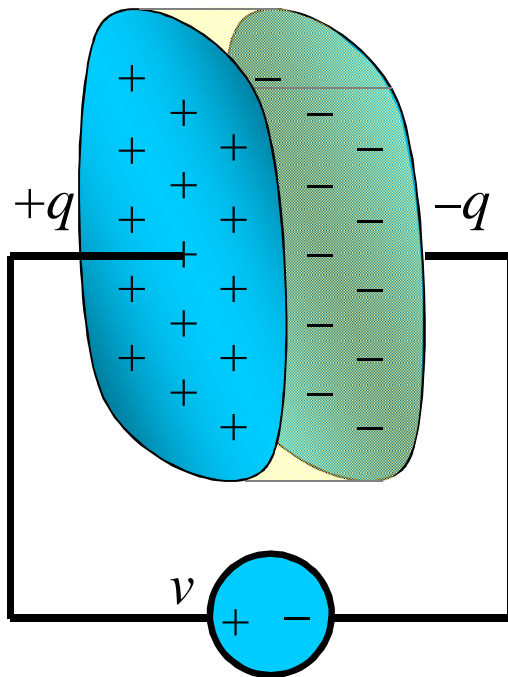
Condensatori e induttori

Sommario

- Il Condensatore
- Tensione, corrente, potenza ed energia nel condensatore
- Condensatori in serie e in parallelo
- L'induttore
- Tensione, corrente, potenza ed energia nell'induttore
- Induttori in serie e in parallelo

Condensatore: relazione tensione/carica

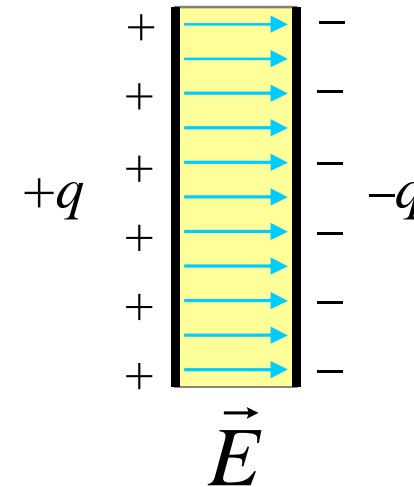
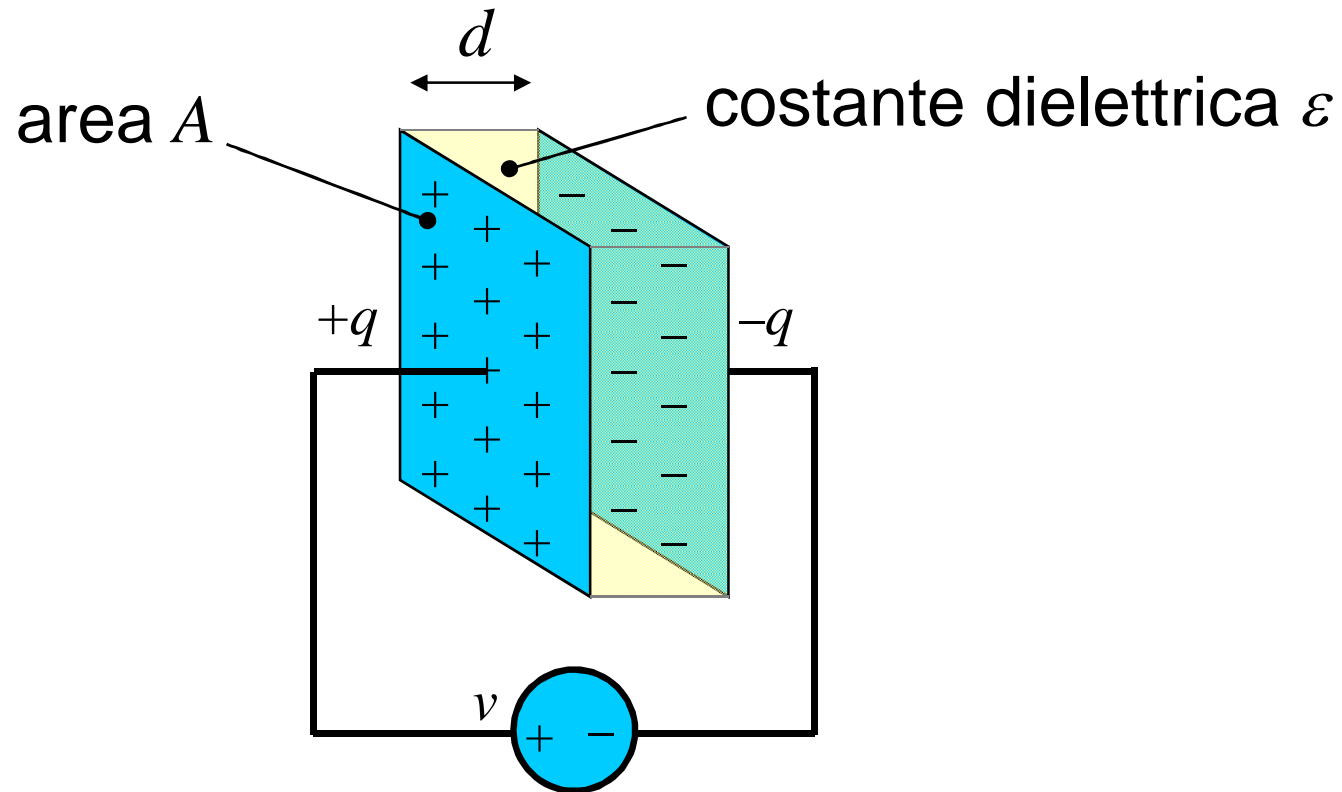
La carica sulle armature è proporzionale alla tensione ai loro capi attraverso il valore di **capacità** del condensatore che si misura in Farad (F)



$$q = C \cdot v$$

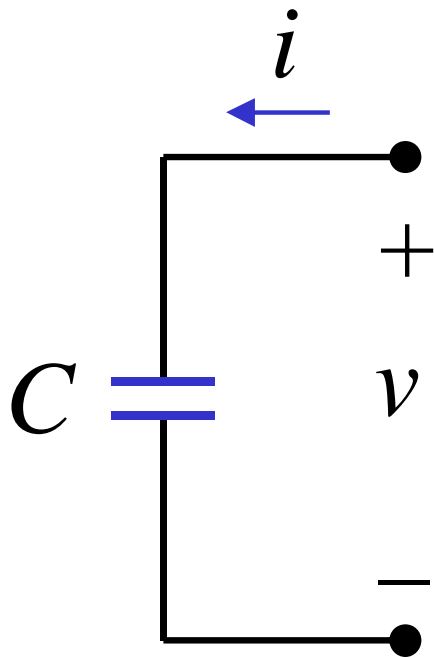
$$1 \text{ F} = 1 \text{ C/V} = 1 \text{ A} \cdot \text{s/V}$$

Condensatore a piani paralleli



$$C = \epsilon \cdot \frac{A}{d}$$

Condensatore: relazione tensione/corrente

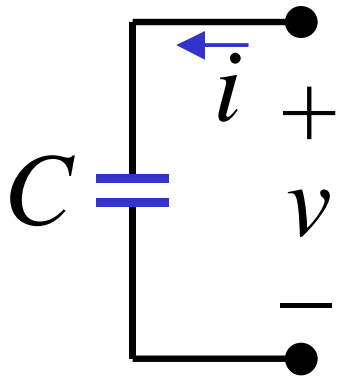


$$i = \frac{dq}{dt} = \frac{d(C \cdot v)}{dt} = C \cdot \frac{dv}{dt}$$

(con la convenzione degli utilizzatori)

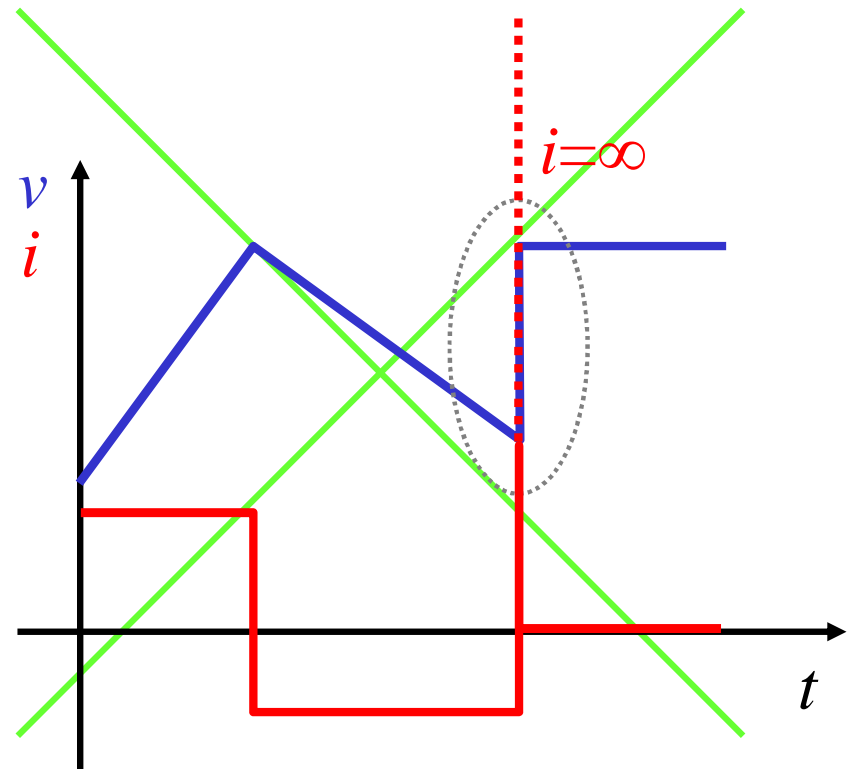
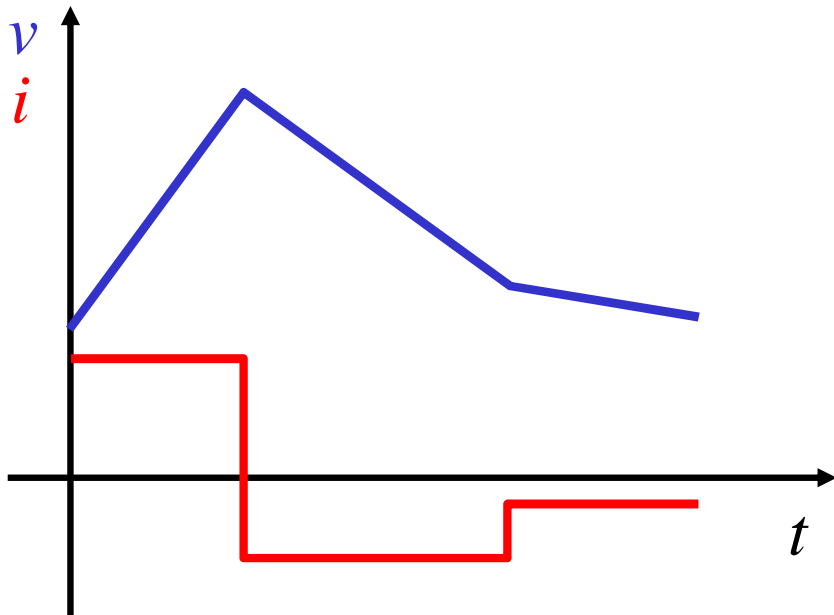
$$v(t) = \frac{1}{C} \int_{t_0}^t i(t) dt + v(t_0)$$

Condensatore: relazione tensione/corrente

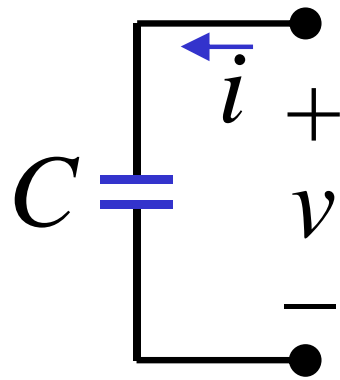


$$i = C \cdot \frac{dv}{dt}$$

$$v(t) = \frac{1}{C} \int_{t_0}^t i(t) dt + v(t_0)$$

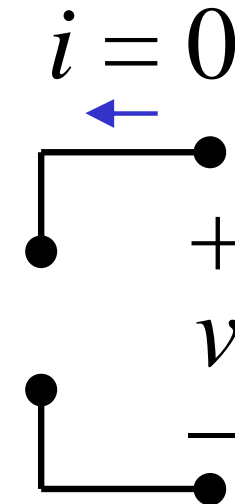
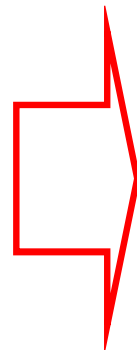
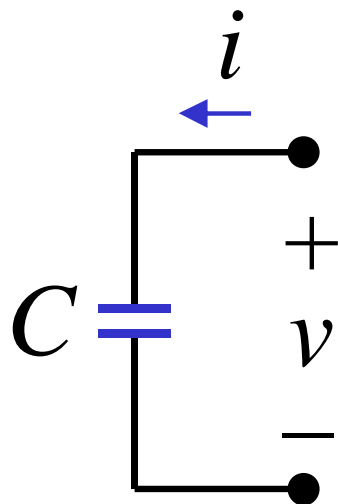


Condensatore in regime stazionario



$$i = C \cdot \frac{dv}{dt}$$

v costante nel tempo $\Rightarrow i = 0$ (circuito aperto)



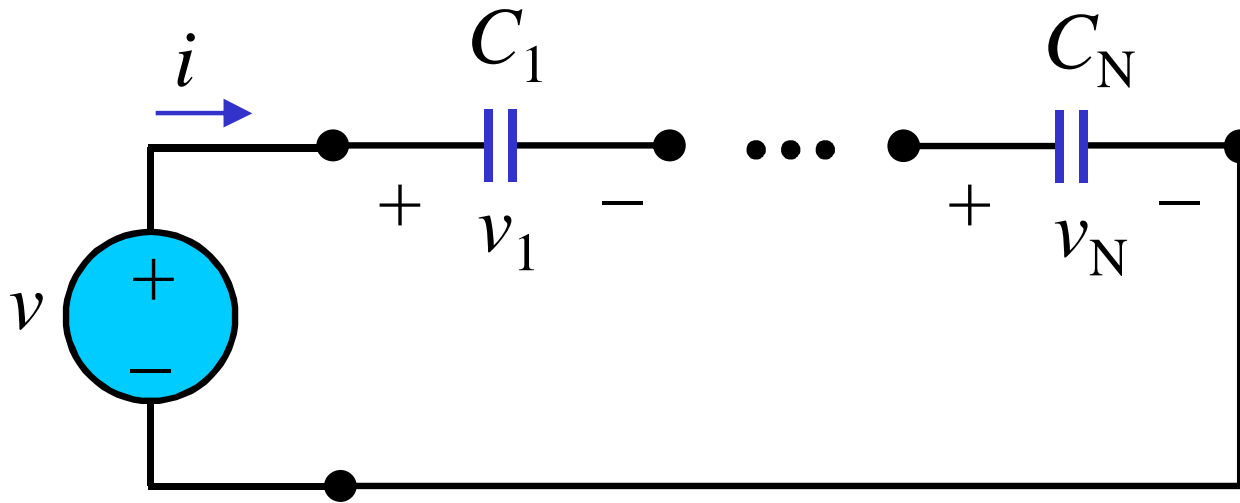
Condensatore: potenza ed energia

$$p = v \cdot i = C \cdot v \cdot \frac{dv}{dt}$$

$$w = \int_{-\infty}^t p(t) dt = C \int_{-\infty}^t v \cdot \frac{dv}{dt} dt = C \int_{v(-\infty)}^{v(t)} v dv =$$

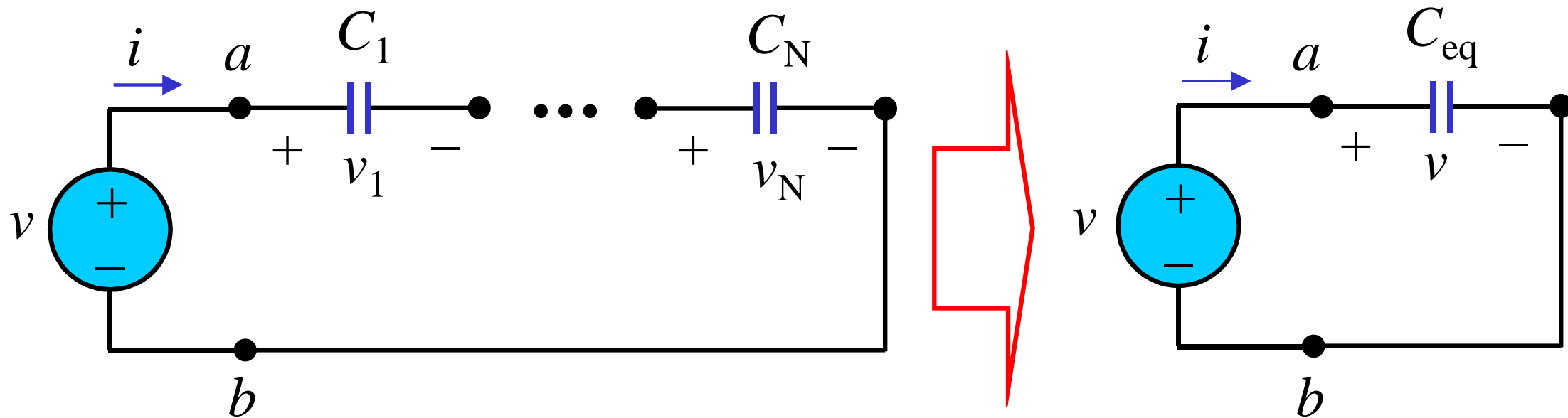
$$= \frac{1}{2} C \cdot v^2 \Big|_0^{v(t)} = \frac{1}{2} C \cdot v^2 = \frac{q^2}{2C}$$

Condensatori in serie



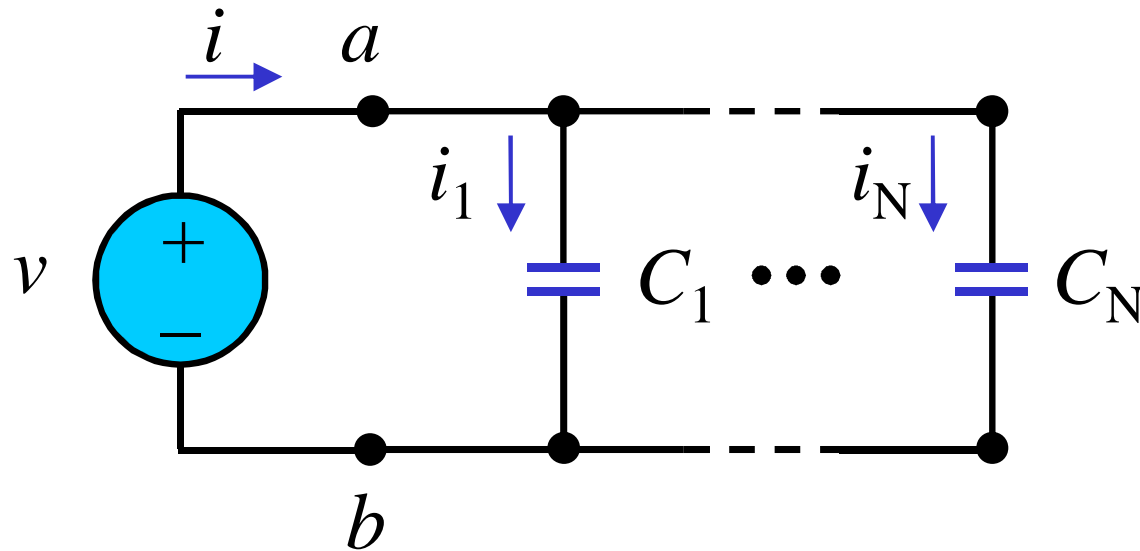
$$\begin{aligned} v &= \frac{1}{C_1} \int_{t_0}^t i dt + v_1(t_0) + \frac{1}{C_2} \int_{t_0}^t i dt + v_2(t_0) + \dots + \frac{1}{C_N} \int_{t_0}^t i dt + v_N(t_0) = \\ &= \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N} \right) \int_{t_0}^t i dt + v_1(t_0) + v_2(t_0) + \dots + v_N(t_0) = \frac{1}{C_{\text{eq}}} \int_{t_0}^t i dt + v(t_0) \end{aligned}$$

Condensatori in serie



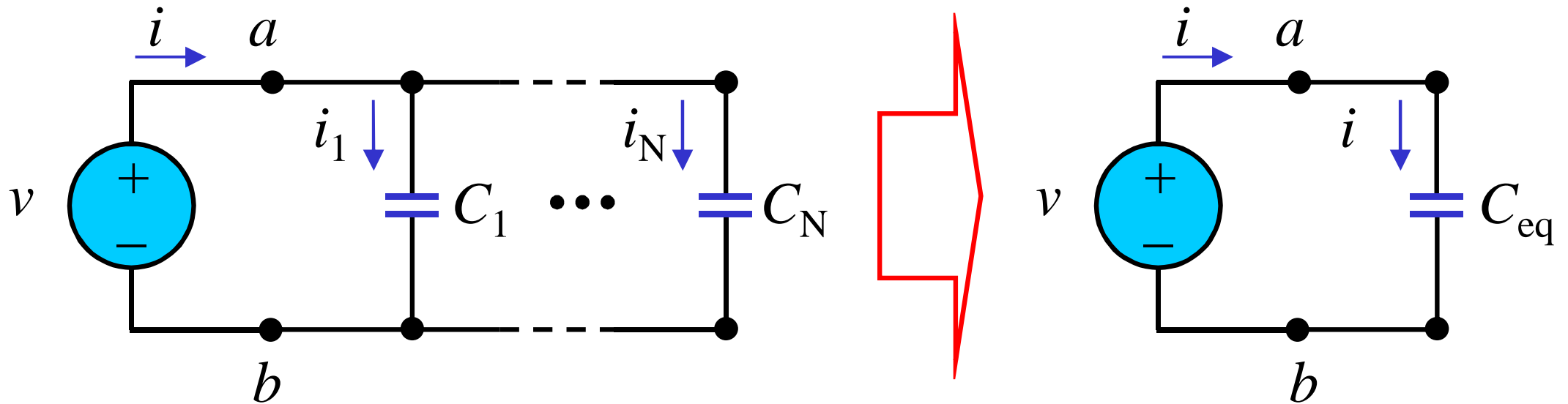
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

Condensatori in parallelo



$$i = C_1 \cdot \frac{dv}{dt} + C_2 \cdot \frac{dv}{dt} + \dots + C_N \cdot \frac{dv}{dt} = (C_1 + C_2 + \dots + C_N) \cdot \frac{dv}{dt} = C_{\text{eq}} \cdot \frac{dv}{dt}$$

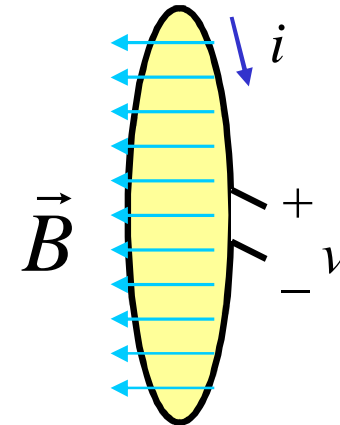
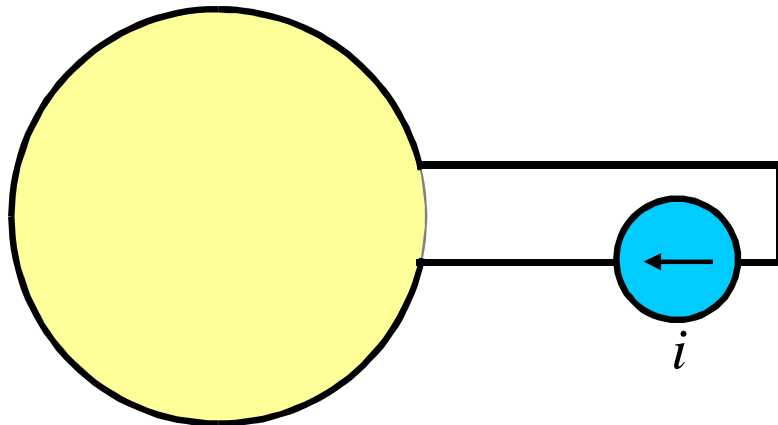
Condensatori in parallelo



$$C_{eq} = C_1 + C_2 + \dots + C_N$$

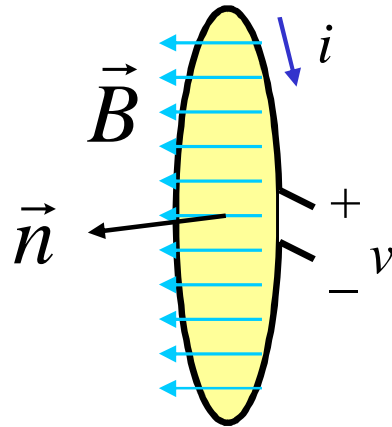
Induttore

Un **induttore** consiste di un filo (tipicamente avvolto in più spire) percorso da corrente ed è in grado di accumulare energia magnetica



Induttore: relazione flusso/corrente

Il flusso del campo magnetico è proporzionale alla corrente attraverso il valore di **induttanza** dell'induttore che si misura in Henry (H)

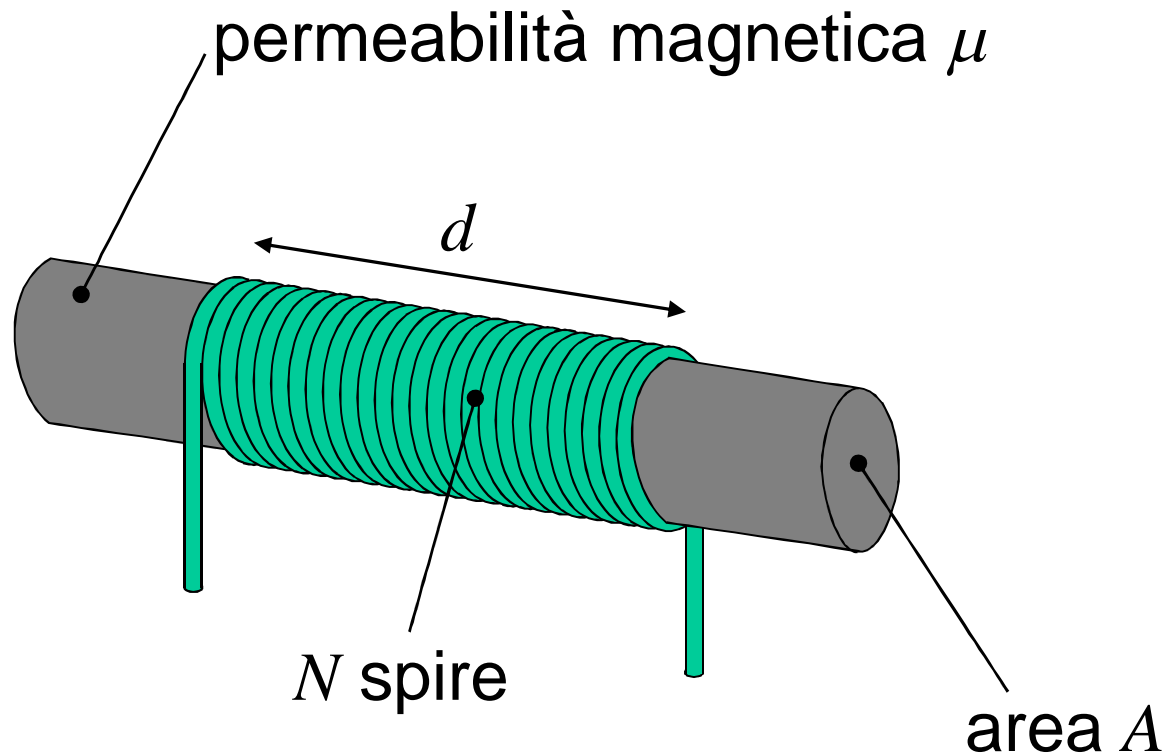


$$\Phi = L \cdot i$$

$$\Phi = \int_S \vec{B} \cdot \vec{n} \, dS$$

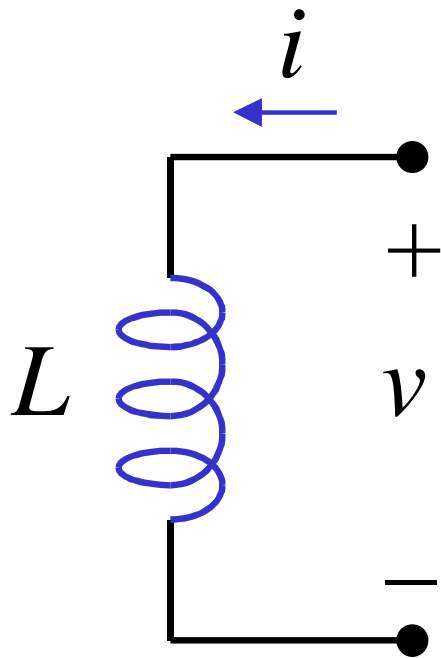
$$1 \text{ H} = 1 \text{ V} \cdot \text{s/A}$$

Induttore a bobina



$$L = \mu \cdot \frac{N^2 A}{d}$$

Induttore: relazione tensione/corrente

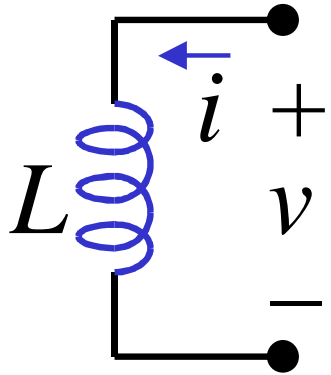


$$v = \frac{d\Phi}{dt} = \frac{d(L \cdot i)}{dt} = L \cdot \frac{di}{dt}$$

(con la convenzione degli utilizzatori)

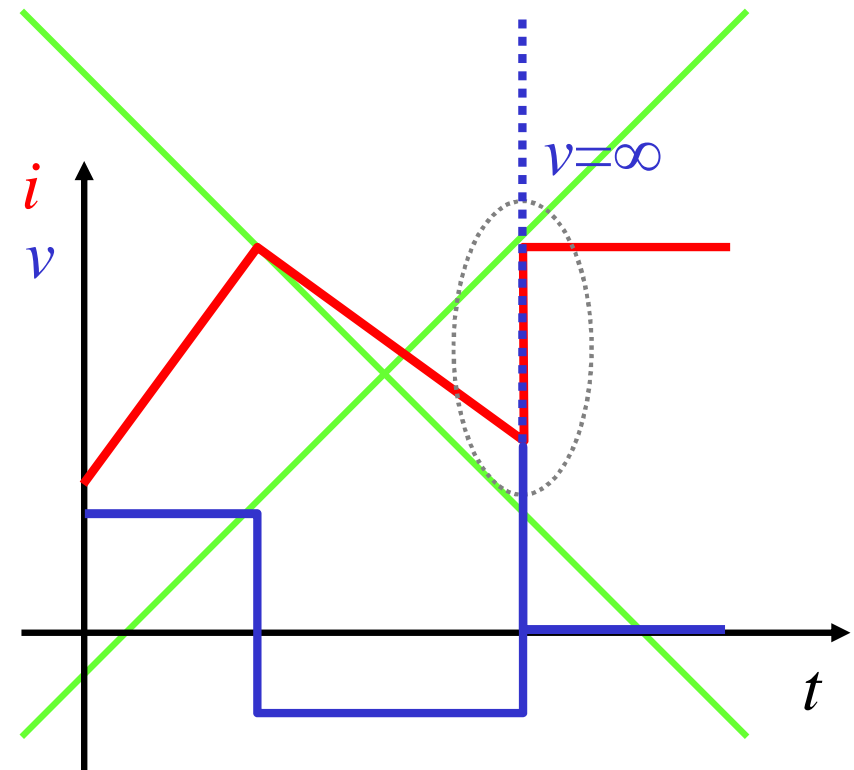
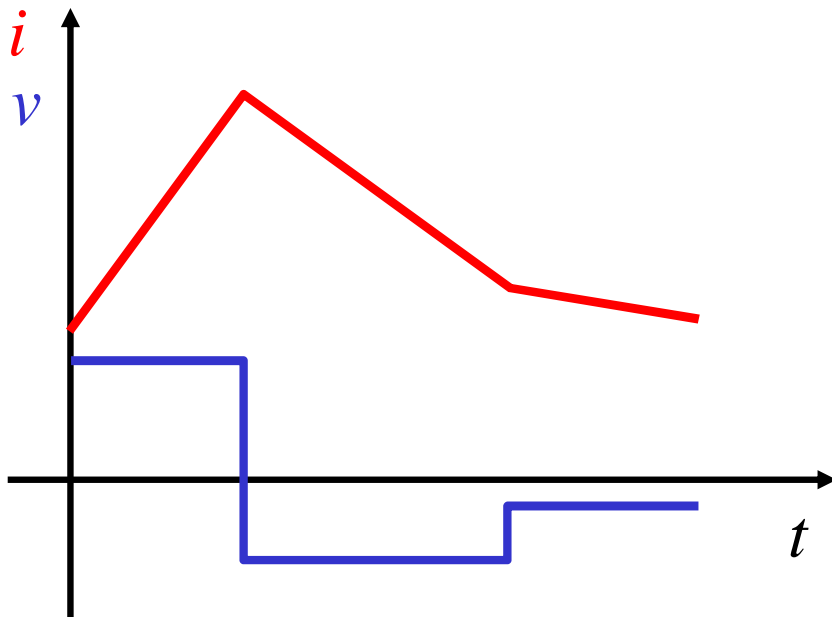
$$i(t) = \frac{1}{L} \int_{t_0}^t v(t) dt + i(t_0)$$

Induttore: relazione tensione/corrente

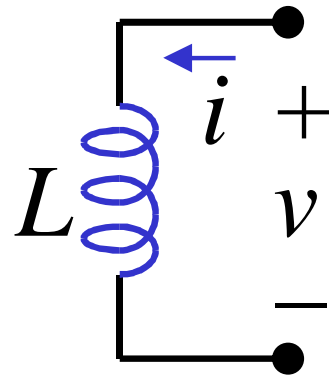


$$v = L \cdot \frac{di}{dt}$$

$$i(t) = \frac{1}{L} \int_{t_0}^t v(t) dt + i(t_0)$$

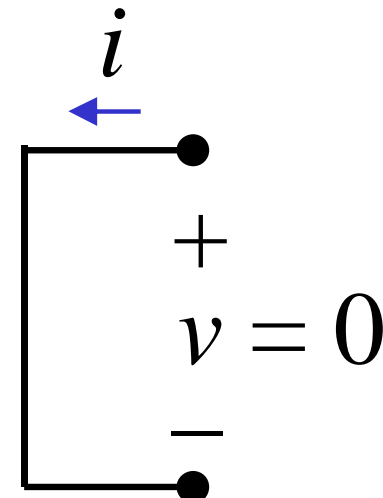
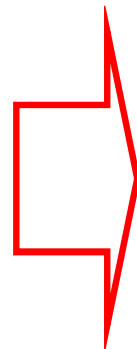
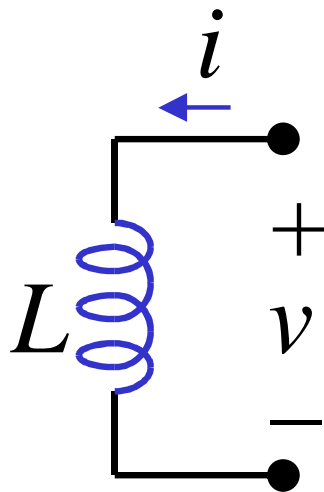


Induttore in regime stazionario



$$v = L \cdot \frac{di}{dt}$$

i costante nel tempo $\Rightarrow v = 0$ (corto circuito)

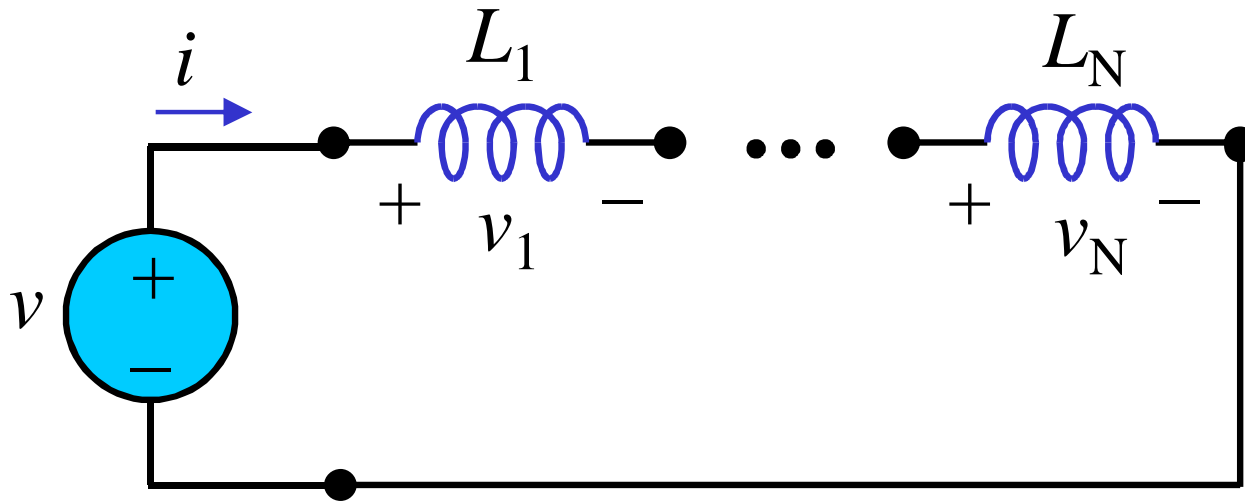


Induttore: potenza ed energia

$$p = v \cdot i = L \cdot \frac{di}{dt} \cdot i$$

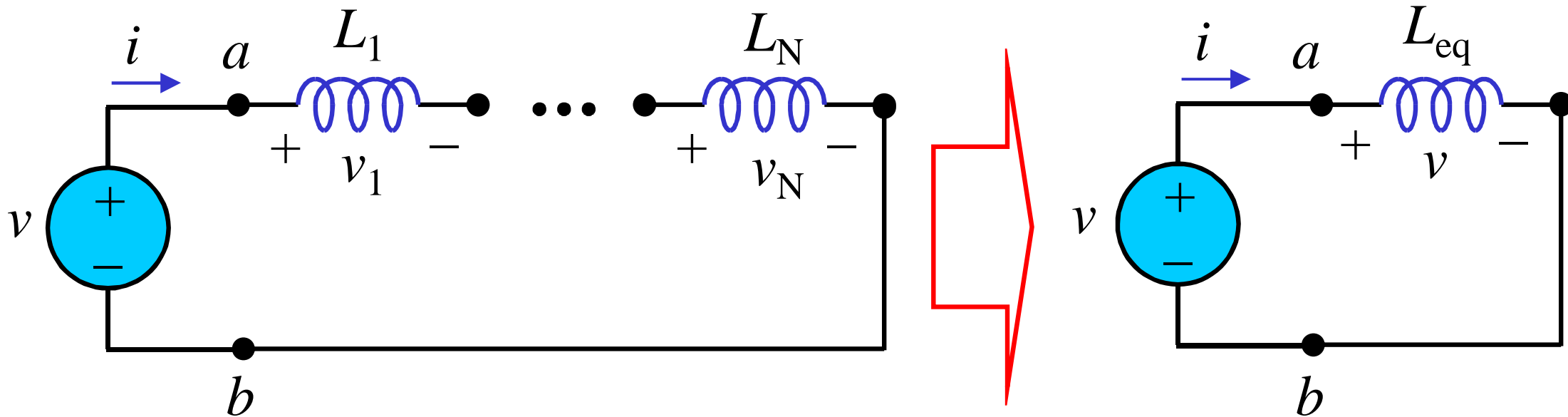
$$\begin{aligned} w &= \int_{-\infty}^t p(t) dt = L \int_{-\infty}^t i \cdot \frac{di}{dt} dt = L \int_{i(-\infty)}^{i(t)} i di = \\ &= \frac{1}{2} L \cdot i^2 \Big|_0^{i(t)} = \frac{1}{2} L \cdot i^2 \end{aligned}$$

Induttori in serie



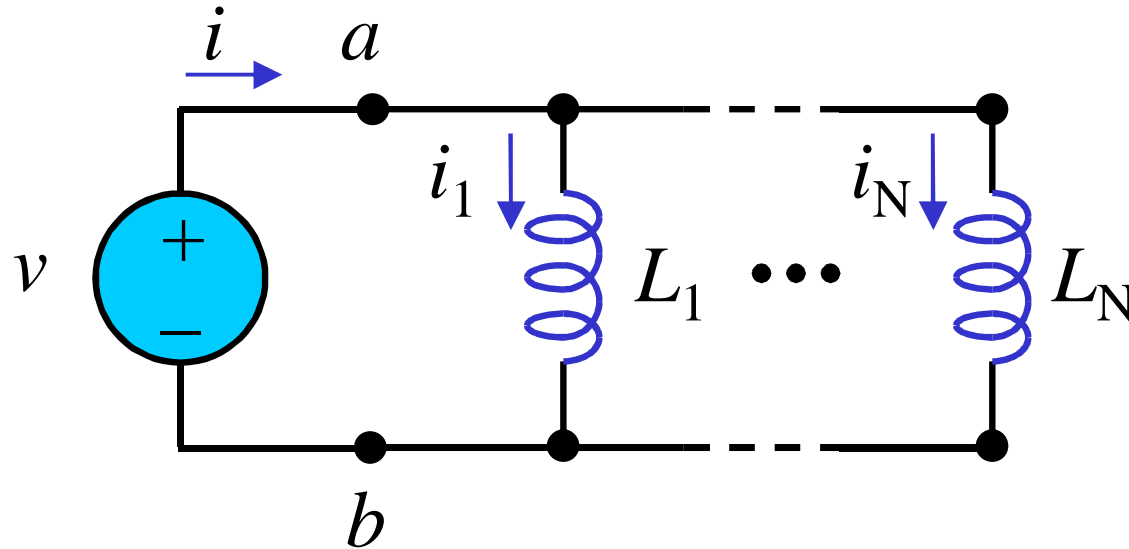
$$v = L_1 \cdot \frac{di}{dt} + L_2 \cdot \frac{di}{dt} + \dots + L_N \cdot \frac{di}{dt} = (L_1 + L_2 + \dots + L_N) \cdot \frac{di}{dt} = L_{\text{eq}} \cdot \frac{di}{dt}$$

Induttori in serie



$$L_{eq} = L_1 + L_2 + \dots + L_N$$

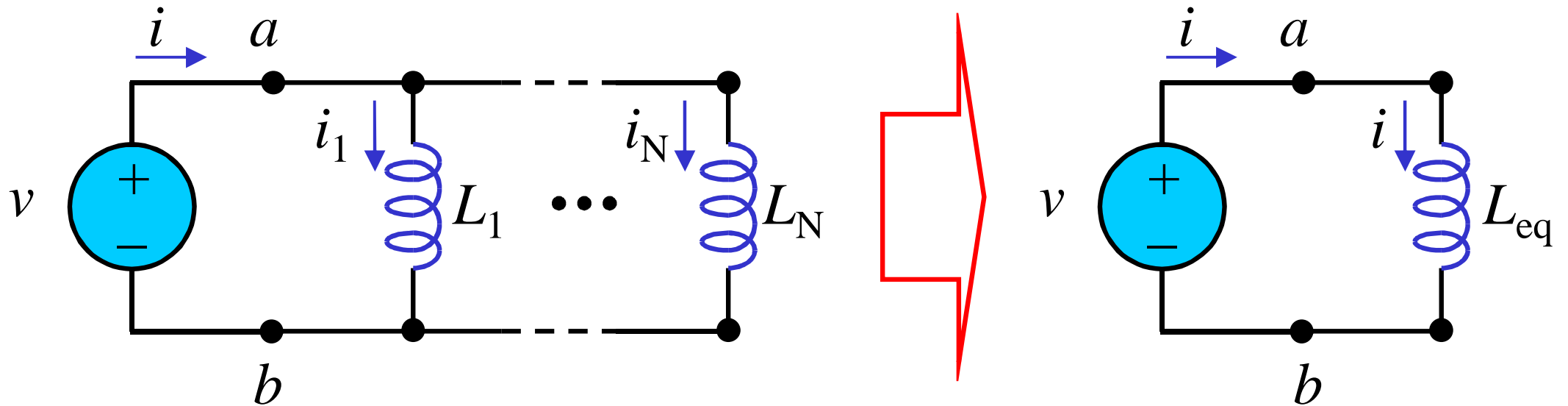
Induttori in parallelo



$$i = \frac{1}{L_1} \int_{t_0}^t v dt + i_1(t_0) + \frac{1}{L_2} \int_{t_0}^t v dt + i_2(t_0) + \dots + \frac{1}{L_N} \int_{t_0}^t v dt + i_N(t_0) =$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N} \right) \int_{t_0}^t v dt + i_1(t_0) + i_2(t_0) + \dots + i_N(t_0) = \frac{1}{L_{\text{eq}}} \int_{t_0}^t v dt + i(t_0)$$

Induttori in parallelo



$$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N}$$