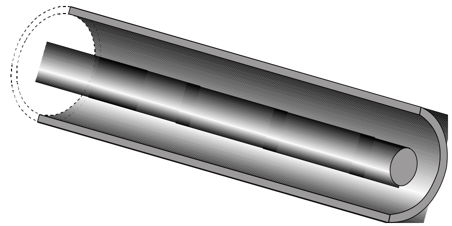
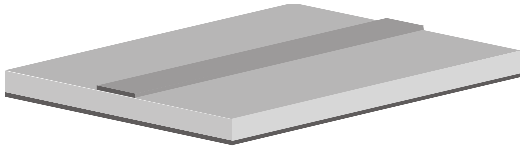
Propagazione guidata

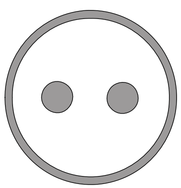
linea bifilare



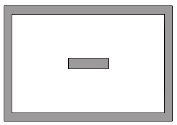
cavo coassiale



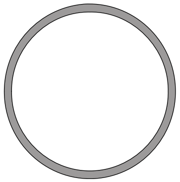
linea in microstriscia



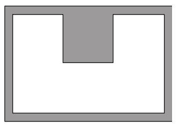
linea bifilare schermata



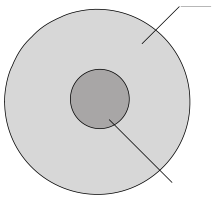
stripline



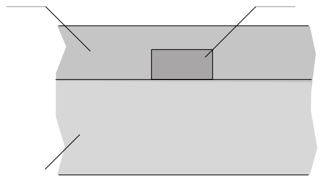
guida circolare



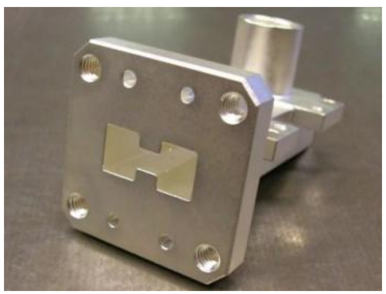
guida rettangolare ribassata (ridge)

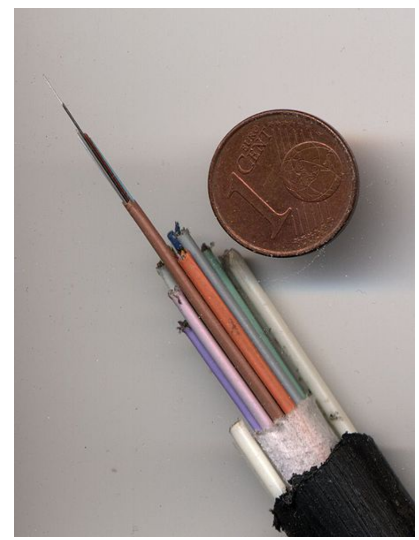


fibra ottica a salto d'indice



fibra ottica integrata





Potenziali di Hertz-Debye

$$\vec{E} = \frac{\nabla \times \nabla \times \vec{u}\,\Psi'}{j\omega\epsilon} - \nabla \times \vec{u}\,\Psi''$$

$$\vec{H} = \nabla \times \vec{u}\,\Psi' + \frac{\nabla \times \nabla \times \vec{u}\,\Psi''}{j\omega\mu}$$

$$\nabla^2\Psi + k^2\Psi = 0$$

$$k^2 = \omega^2\epsilon\mu.$$

- condizione di parete elettrica sui conduttori
- continuità delle componenti tangenziali dei campi sulle interfacce con altri dielettrici
- opportune condizioni all'infinito, nel caso di strutture non schermate

$$\Psi' = \Phi'(\vec{\rho})\, I(z) \qquad\qquad \Psi'' = \Phi''(\vec{\rho})\, V(z)$$

$$\begin{aligned}
\vec{E}_{\mathrm{T}} &= \frac{\eta}{jk}\, \nabla_{\mathrm{T}}\, \Phi'(\vec{\rho})\, \frac{\partial I(z)}{\partial z} + \vec{u}_z \times \nabla_{\mathrm{T}}\, \Phi''(\vec{\rho})\, V(z) \\
\vec{H}_{\mathrm{T}} &= -\vec{u}_z \times \nabla_{\mathrm{T}}\, \Phi'(\vec{\rho})\, I(z) + \frac{1}{jk\,\eta}\, \nabla_{\mathrm{T}}\, \Phi''(\vec{\rho})\, \frac{\partial V(z)}{\partial z} \\
E_z &= -\frac{\eta}{jk}\, \nabla_{\mathrm{T}}^2 \Phi'(\vec{\rho})\, I(z) \\
H_z &= -\frac{1}{jk\,\eta}\, \nabla_{\mathrm{T}}^2 \Phi''(\vec{\rho})\, V(z)
\end{aligned}$$

$$\frac{\partial V(z)}{\partial z} = -\gamma\, Z\, I(z) \qquad\qquad \nabla_{\mathrm{T}}^2\, \Phi(\vec{\rho}) + \kappa^2\, \Phi(\vec{\rho}) = 0$$

$$\frac{\partial I(z)}{\partial z} = -\gamma\, \frac{V(z)}{Z} \qquad\qquad \kappa^2 = k^2 + \gamma^2$$

$$\Psi' = \Phi'(\vec{\rho})\, I(z) \qquad \Psi'' = \Phi''(\vec{\rho})\, V(z)$$

$$\frac{\partial I(z)}{\partial z} = -\gamma\,\frac{V(z)}{Z} \qquad \frac{\partial V(z)}{\partial z} = -\gamma\, Z\, I(z)$$

$$\begin{aligned}
\vec{E}_{\mathrm{T}} &= \left(-\frac{\eta\,\gamma}{jkZ}\,\nabla_{\mathrm{T}}\,\Phi'(\vec{\rho}) + \vec{u}_z \times \nabla_{\mathrm{T}}\,\Phi''(\vec{\rho})\right) V(z) \\
\vec{H}_{\mathrm{T}} &= \left(-\frac{\gamma Z}{jk\,\eta}\,\nabla_{\mathrm{T}}\,\Phi''(\vec{\rho}) - \vec{u}_z \times \nabla_{\mathrm{T}}\,\Phi'(\vec{\rho})\right) I(z) \\
E_z &= \frac{\eta\,\kappa^2}{jk}\,\Phi'(\vec{\rho})\, I(z) \\
H_z &= \frac{\kappa^2}{jk\,\eta}\,\Phi''(\vec{\rho})\, V(z)
\end{aligned}$$

$$\nabla_{\mathrm{T}}^2\,\Phi(\vec{\rho}) + \kappa^2\,\Phi(\vec{\rho}) = 0$$

$$\kappa^2 = k^2 + \gamma^2$$

guide d'onda

modi TM		modi TE		
$\nabla_{\mathrm{T}}^2\, \Phi_i' + \kappa_i'^2\, \Phi_i' = 0$		$\nabla_{\mathrm{T}}^2\, \Phi_i'' + \kappa_i''^2\, \Phi_i'' = 0$		equazione
$\Phi_i'(\vec{\rho}) = 0$	$\vec{\rho} \in C$	$\dfrac{\partial \Phi_i''(\vec{\rho})}{\partial n} = 0$	$\vec{\rho} \in C$	condizione al contorno
$\int_S \Phi_i'^2 ds = 1$		$\int_S \Phi_i''^2 ds = 1$		condizione di normalizzazione
$\int_S \Phi_i'\, \Phi_j'\, ds = 0$	$(i \neq j)$	$\int_S \Phi_i''\, \Phi_j''\, ds = 0$	$(i \neq j)$	ortogonalità
$\gamma_i' = \sqrt{\kappa_i'^2 - k^2}$		$\gamma_i'' = \sqrt{\kappa_i''^2 - k^2}$		costante di propagazione
$\vec{e}_i' = -\dfrac{\nabla_{\mathrm{T}}\Phi_i'}{\kappa_i'}$		$\vec{h}_i'' = -\dfrac{\nabla_{\mathrm{T}}\Phi_i''}{\kappa_i''}$		vettori modali
$\vec{h}_i' = \vec{u}_z \times \vec{e}_i'$		$\vec{e}_i'' = -\vec{u}_z \times \vec{h}_i''$		
$Z_i' = \eta\, \dfrac{\gamma_i'}{jk}$		$Z_i'' = \eta\, \dfrac{jk}{\gamma_i''}$		impedenza modale
$\vec{E}_i' = \vec{e}_i'\, V_i' - j\, \vec{u}_z \dfrac{\kappa_i'}{k}\, \Phi_i'\, \eta\, I_i'$		$\vec{E}_i'' = \vec{e}_i''\, V_i''$		campi modali
$\vec{H}_i' = \vec{h}_i'\, I_i'$		$\vec{H}_i'' = \vec{h}_i''\, I_i'' - j\, \vec{u}_z \dfrac{\kappa_i''}{k}\, \Phi_i''\, \dfrac{V_i''}{\eta}$		

Si nota che la condizione di normalizzazione implica che i potenziali Φ_i, sia TE che TM, siano dimensionalmente uguali all'inverso di una lunghezza, così che risulta adimensionale il rapporto Φ_i/κ_i.

$\omega < \omega_c \equiv \lambda > \lambda_c$	$\omega > \omega_c \equiv \lambda < \lambda_c$
$\alpha = \mathrm{Re}(\gamma) = \frac{2\pi}{\lambda_c}\sqrt{1 - \frac{\lambda_c^2}{\lambda^2}} = \frac{\omega_c}{v}\sqrt{1 - \frac{\omega^2}{\omega_c^2}}$	$\alpha = \mathrm{Re}(\gamma) = 0$
$\beta = \mathrm{Im}(\gamma) = 0$	$\beta = \mathrm{Im}(\gamma) = \frac{2\pi}{\lambda}\sqrt{1 - \frac{\lambda^2}{\lambda_c^2}} = \frac{\omega}{v}\sqrt{1 - \frac{\omega_c^2}{\omega^2}}$
$Z' = -j\eta\,\frac{\alpha}{k} = -j\,\eta\sqrt{\omega_c^2/\omega^2 - 1}$	$Z' = \eta\,\frac{\beta}{k} = \eta\,\frac{\lambda}{\lambda_g} = \eta\,\sqrt{1 - \omega_c^2/\omega^2} \;<\; \eta$
$Z'' = j\eta\,\frac{k}{\alpha} = j\,\frac{\eta}{\sqrt{\omega_c^2/\omega^2 - 1}}$	$Z'' = \eta\,\frac{k}{\beta} = \eta\,\frac{\lambda_g}{\lambda} = \frac{\eta}{\sqrt{1 - \omega_c^2/\omega^2}} \;>\; \eta$
Z immaginarie	Z reali

$$
\begin{aligned}
\vec{E}(\vec{\rho},z) &= \sum_i \left(\vec{e}_i{}'(\vec{\rho})\, V_i{}'(z) - j\, \vec{u}_z \frac{\kappa_i'}{k}\; \Phi_i'(\vec{\rho})\, \eta\, I_i{}'(z) \right) + \sum_i \vec{e}_i{}''(\vec{\rho})\, V_i{}''(z) \\
\vec{H}(\vec{\rho},z) &= \sum_i \vec{h}_i{}'(\vec{\rho})\, I_i{}'(z) + \sum_i \left(\vec{h}_i{}''(\vec{\rho})\, I_i{}''(z) - j\, \vec{u}_z \frac{\kappa_i''}{k}\; \Phi_i''(\vec{\rho})\, \frac{V_i{}''(z)}{\eta} \right)
\end{aligned}
$$

$$
\begin{aligned}
V_i(z) &= V_i^+\, e^{-\gamma_i z} + V_i^-\, e^{\gamma_i z} \\
I_i(z) &= \frac{V_i^+\, e^{-\gamma_i z} - V_i^-\, e^{\gamma_i z}}{Z_i}
\end{aligned}
$$

$$\frac{1}{2}\int_S \mathrm{Re}\left(\vec{E}\times\vec{H}^*\right)\cdot\vec{u}_z\,ds = \frac{1}{2}\sum_i \mathrm{Re}(V_i{}'\,I_i'{}^*) + \frac{1}{2}\sum_i \mathrm{Re}(V_i{}''\,I_i''{}^*)$$

$$\int_S \vec{e}_i\times\vec{h}_i\cdot\vec{u}_z\,ds = 1$$

$$\int_S \vec{e}_i\times\vec{h}_j\cdot\vec{u}_z\,ds = 0 \qquad (i\neq j)$$

$$P_i = \frac{|V_i^+|^2}{2\,Z_i} - \frac{|V_i^-|^2}{2\,Z_i}$$

se il modo *i* si propaga
(Z_i reale)

$$P_i = \frac{\mathrm{Im}(V_i^+\,V_i^{-\,*})}{\mathrm{Im}(Z_i)}$$

Se il modo *i* è evanescente
(Z_i immaginaria)

guida rettangolare

modi TM_{mn} $(m, n = 1, 2, \ldots)$	modi TE_{mn} $(m, n = 0, 1, 2, \ldots$ escluso $m = n = 0)$

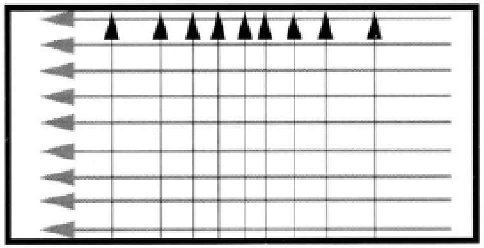
$$\kappa_{mn} = \pi \sqrt{(m/a)^2 + (n/b)^2}$$

$$\lambda_{mn} = \frac{2\,\pi}{\kappa_{mn}} = 2\Big/\sqrt{(m/a)^2 + (n/b)^2}$$

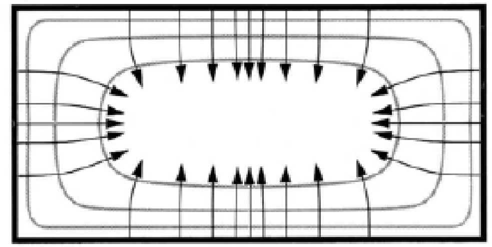
$\Phi'_{mn} = \dfrac{2}{\sqrt{ab}}\, S_x\, S_y$	$\Phi''_{mn} = \dfrac{2}{\sqrt{\chi_{mn}\, ab}}\, C_x\, C_y$
$\vec{e}'_{mn} = \dfrac{\lambda_{mn}}{\sqrt{ab}} \left(-\vec{u}_x\, \frac{m}{a}\, C_x\, S_y - \vec{u}_y\, \frac{n}{b}\, S_x\, C_y\right)$	$\vec{e}''_{mn} = \dfrac{\lambda_{mn}}{\sqrt{\chi_{mn}\, ab}} \left(\vec{u}_x\, \frac{n}{b}\, C_x\, S_y - \vec{u}_y\, \frac{m}{a}\, S_x\, C_y\right)$
$\vec{h}'_{mn} = \dfrac{\lambda_{mn}}{\sqrt{ab}} \left(\vec{u}_x\, \frac{n}{b}\, S_x\, C_y - \vec{u}_y\, \frac{m}{a}\, C_x\, S_y\right)$	$\vec{h}''_{mn} = \dfrac{\lambda_{mn}}{\sqrt{\chi_{mn}\, ab}} \left(\vec{u}_x\, \frac{m}{a}\, S_x\, C_y + \vec{u}_y\, \frac{n}{b}\, C_x\, S_y\right)$

$$S_x = \sin\frac{m\pi x}{a} \qquad S_y = \sin\frac{n\pi y}{b} \qquad C_x = \cos\frac{m\pi x}{a} \qquad C_y = \cos\frac{n\pi y}{b}$$

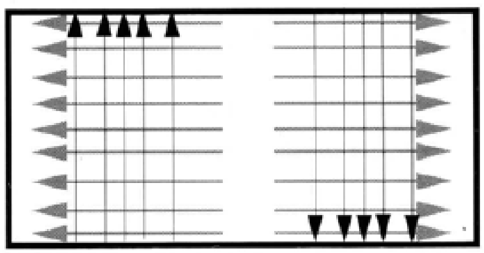
$$\chi_{mn} = \begin{cases} 2 & \text{se } m = 0 \text{ o } n = 0 \\ 1 & \text{se } m \neq 0 \text{ e } n \neq 0 \end{cases}$$



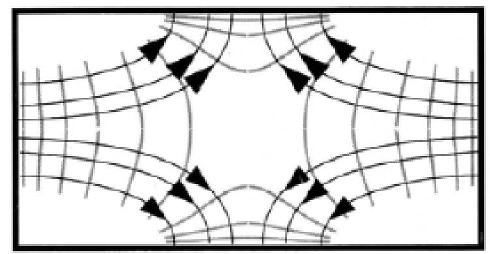
TE_{10}



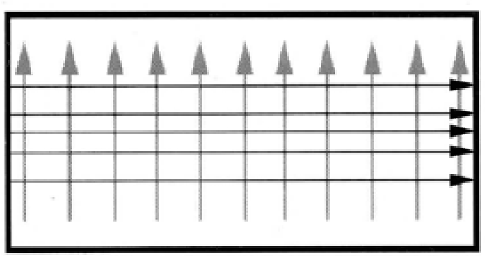
TM_{11}



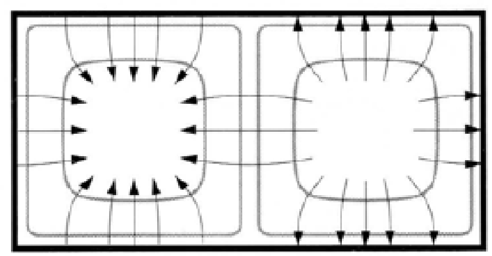
TE_{20}



TE_{11}

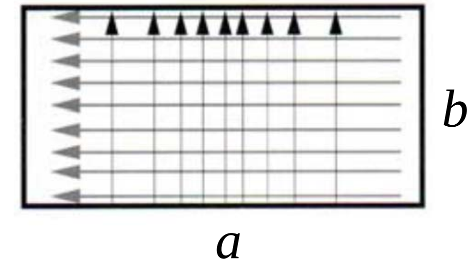


TE_{01}



TM_{21}

modo TE_{10} in guida rettangolare



$$\kappa''_{10} = \pi/a \quad \lambda''_{10} = 2\,a \quad f''_{10} = v/(2\,a)$$

$$\Phi''_{10} = \sqrt{\frac{2}{ab}}\ \cos\frac{\pi x}{a}$$

potenziale

$$\vec{e}''_{10} = -\sqrt{\frac{2}{ab}}\ \vec{u}_y\ \sin\frac{\pi x}{a}$$

$$\vec{h}''_{10} = \sqrt{\frac{2}{ab}}\ \vec{u}_x\ \sin\frac{\pi x}{a}$$

vettori modali

campi modali

$$\vec{E}''_{10} = -\sqrt{\frac{2}{ab}}\,\vec{u}_y\,\sin\frac{\pi x}{a}\,V''_{10}(z)$$

$$\vec{H}''_{10} = \sqrt{\frac{2}{ab}}\left(\vec{u}_x\ \sin\frac{\pi x}{a}\ I''_{10}(z) - j\ \vec{u}_z\cos\frac{\pi x}{a}\ \frac{\lambda}{2a\,\eta}\ V''_{10}(z)\right)$$

guida d'onda circolare

modi $\mathrm{TM}_{mn}^{(e)}$ $(m = 0, 1, 2, \ldots; n = 1, 2, \ldots)$

$\kappa'_{mn} = \dfrac{x_{mn}}{a}$ x_{mn} è la n-esima radice dell'equazione $J_m(x) = 0$ (zero escluso) $\lambda'_{mn} = \dfrac{2\pi a}{x_{mn}}$

$$\Phi_{mn}^{\prime(e)} = F'_{mn}\, J_m(\kappa'_{mn} r)\, \cos m\varphi$$

$$\vec{e}_{mn}^{\,\prime(e)} = -F'_{mn}\left(\vec{u}_r\, J'_m(\kappa'_{mn} r)\, \cos m\varphi - \vec{u}_\varphi\, \frac{m\, J_m(\kappa'_{mn} r)}{\kappa'_{mn} r}\, \sin m\varphi\right)$$

$$\vec{h}_{mn}^{\,\prime(e)} = -F'_{mn}\left(\vec{u}_r \frac{m\, J_m(\kappa'_{mn} r)}{\kappa'_{mn} r}\, \sin m\varphi + \vec{u}_\varphi\, J'_m(\kappa'_{mn} r)\, \cos m\varphi\right)$$

$$F'_{mn} = \frac{\chi_m}{a\sqrt{\pi}\, J_{m+1}(x_{mn})} \qquad J'_m(\kappa'_{mn} r) = \left.\frac{dJ_m(x)}{d\,x}\right|_{x=\kappa'_{mn} r} \qquad \chi_m = \begin{cases} 1 & \text{se } m = 0 \\ \sqrt{2} & \text{se } m \neq 0 \end{cases}$$

modi $\mathrm{TE}_{mn}^{(e)}$ $(m = 0, 1, 2, \ldots; n = 1, 2, \ldots)$

$\kappa''_{mn} = \dfrac{x'_{mn}}{a}$ x'_{mn} è la n-esima radice dell'equazione $J'_m(x) = 0$ (zero escluso) $\lambda''_{mn} = \dfrac{2\pi a}{x'_{mn}}$

$$\Phi_{mn}^{\prime\prime(e)} = F''_{mn}\, J_m(\kappa''_{mn} r)\, \cos m\varphi$$

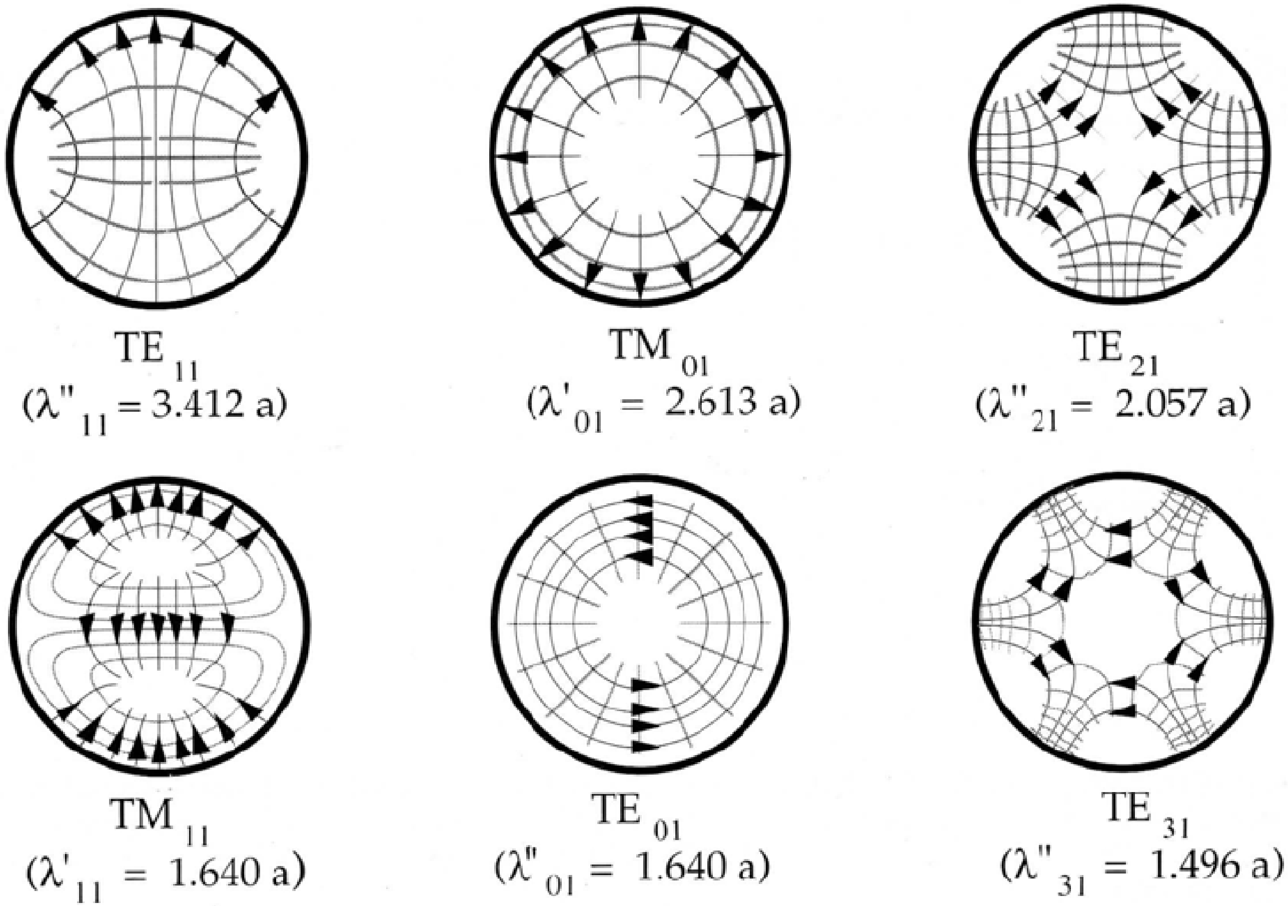
$$\vec{e}_{mn}^{\,\prime\prime(e)} = F''_{mn}\left(\vec{u}_r\, \frac{m\, J_m(\kappa''_{mn} r)}{\kappa''_{mn} r}\, \sin m\varphi + \vec{u}_\varphi\, J'_m(\kappa''_{mn} r)\, \cos m\varphi\right)$$

$$\vec{h}_{mn}^{\,\prime\prime(e)} = F''_{mn}\left(-\vec{u}_r\, J'_m(\kappa''_{mn} r)\, \cos m\varphi + \vec{u}_\varphi\, \frac{m\, J_m(\kappa''_{mn} r)}{\kappa''_{mn} r}\, \sin m\varphi\right)$$

$$F''_{mn} = \frac{\chi\, \kappa''_{mn}}{J_m(x'_{mn})\sqrt{\pi\, (x'^{\,2}_{mn} - m^2)}}$$

modi $\mathrm{TM}_{mn}^{(o)}$ e $\mathrm{TE}_{mn}^{(o)}$ $(m = 1, 2, \ldots; n = 1, 2, \ldots)$

le espressioni dei potenziali e dei vettori modali relativi ai modi dispari si deducono dalle corrispondenti espressioni relative ai modi pari cambiando $\cos \to \sin$ e $\sin \to -\cos$.


TE 11
(λ" 11 = 3.412 a)
TM 01
(λ' 01 = 2.613 a)
TE 21
(λ" 21 = 2.057 a)
TM 11
(λ' 11 = 1.640 a)
TE 01
(λ" 01 = 1.640 a)
TE 31
(λ" 31 = 1.496 a)

modo TE_{11} in guida circolare

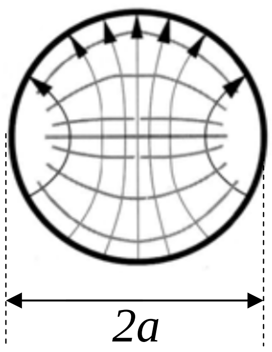
modo $\mathrm{TE}_{11}^{(e)}$	$\kappa''_{11} = x'_{11}\,/\,a \;\approx\; 1.841\,/\,a$	$\lambda''_{11} = 2\pi a\,/\,x'_{11} \;\approx 3.412\,a$

$$\Phi''^{(e)}_{11} = F''_{11}\; J_1(\kappa''_{11} r)\; \cos\varphi$$

$$\vec{e}\,''^{(e)}_{11} = F''_{11} \left(\vec{u}_r \, \frac{J_1(\kappa''_{11} r)}{\kappa''_{11} r} \; \sin\varphi + \vec{u}_\varphi \; J'_1(\kappa''_{11} r)\; \cos\varphi \right)$$

$$\vec{h}\,''^{(e)}_{11} = F''_{11} \left(-\vec{u}_r \; J'_m(\kappa''_{11} r)\; \cos\varphi + \vec{u}_\varphi \, \frac{J_1(\kappa''_{11} r)}{\kappa''_{11} r} \; \sin\varphi \right)$$

$$F''_{11} = \frac{\sqrt{2}\;\kappa''_{11}}{J_1(x'_{11})\sqrt{\pi\,(x'^{\,2}_{11} - 1)}} \;\approx\; \frac{1.633}{a}$$

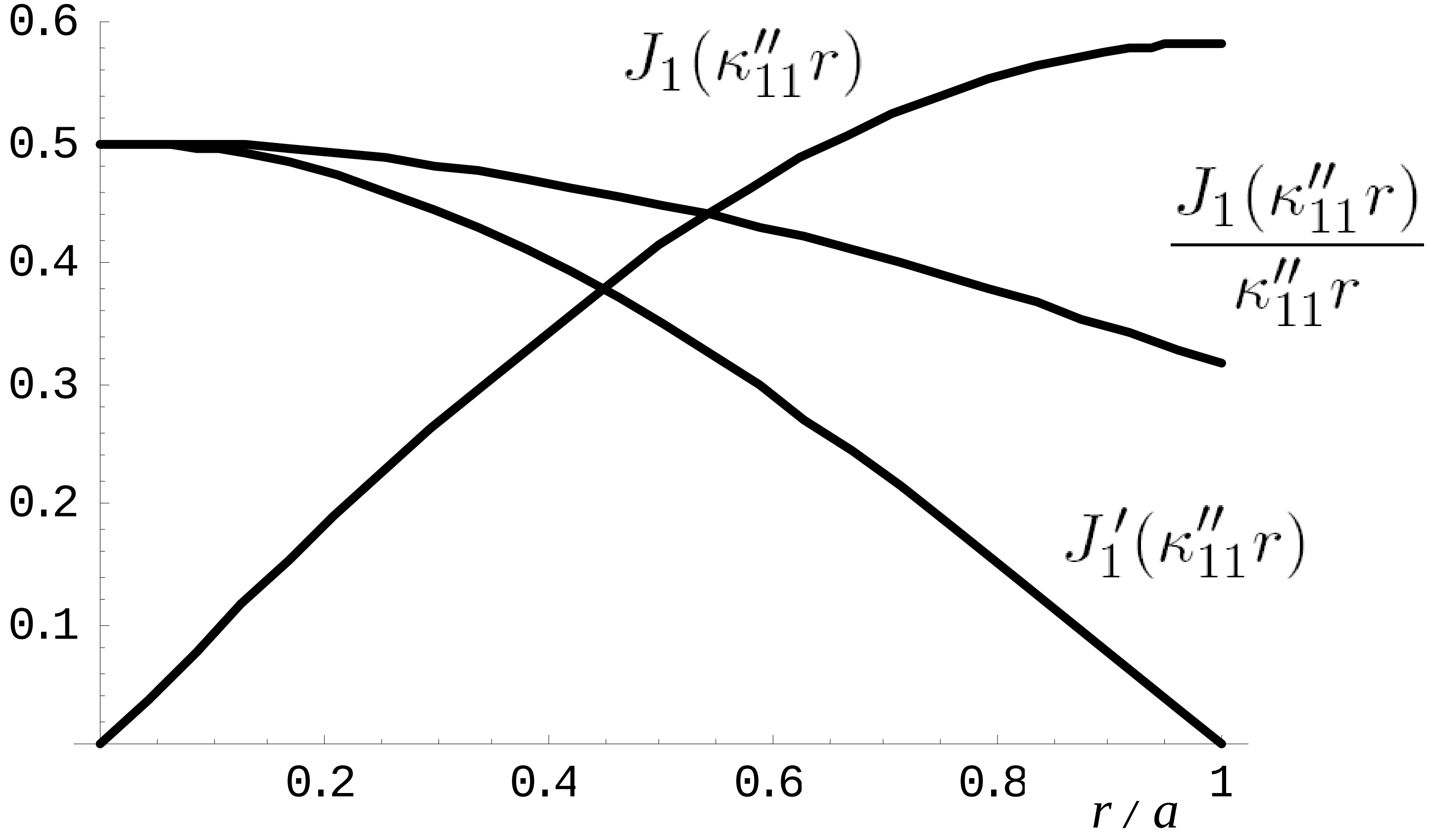


le espressioni del potenziale e dei vettori modali relativi al modo dispari si deducono dalle precedenti cambiando $\cos\varphi \rightarrow \sin\varphi$ e $\sin\varphi \rightarrow -\cos\varphi$.

campi modali

$$\vec{E}''_{11} \;=\; \vec{e}\,''^{(e)}_{11}\, V''^{(e)}_{11}(z) + \vec{e}\,''^{(o)}_{11}\, V''^{(o)}_{11}(z)$$

$$\vec{H}''_{11} \;=\; \vec{h}\,''^{(e)}_{11}\, I''^{(e)}_{11}(z) + \vec{h}\,''^{(o)}_{11}\; I''^{(o)}_{11}(z) - j\,\vec{u}_z \frac{\lambda}{\lambda''_{11}} \left(\Phi''^{(e)}_{11} \,\frac{V''^{(e)}_{11}(z)}{\eta} + \Phi''^{(o)}_{11} \,\frac{V''^{(o)}_{11}(z)}{\eta} \right)$$


0.6
0.5
0.4
0.3
0.2
0.1
0.2
0.4
0.6
0.8
1
r / a
$J_1(\kappa''_{11} r)$
$\frac{J_1(\kappa''_{11} r)}{\kappa''_{11} r}$
$J'_1(\kappa''_{11} r)$

modi TEM

$\nabla_{\mathrm{T}}^2\, \Phi_i^{\circ} = 0$		equazione
$\Phi_i^{\circ}(\vec{\rho}) = 0$	$\vec{\rho} \in C_0$	condizione al contorno
$\Phi_i^{\circ}(\vec{\rho}) = \mathrm{cost}_i^{(n)}$	$\vec{\rho} \in C_n$	
$\gamma_i^{\circ} = jk$		costante di propagazione
$\vec{e}_i^{\,\circ} = -\nabla_{\mathrm{T}} \Phi_i^{\circ}$		vettori modali
$\vec{h}_i^{\,\circ} = \vec{u}_z \times \vec{e}_i^{\,\circ}$		
$Z_i^{\circ} = \eta$		impedenza modale
$\vec{E}_i^{\circ} = \vec{e}_i^{\,\circ}\, V_i^{\circ}$		campi modali
$\vec{H}_i^{\circ} = \vec{h}_i^{\,\circ}\, I_i^{\circ}$		

$$V_i^{(n)} = \int_{\vec{\rho}_0}^{\vec{\rho}_n} \vec{E}_i^{\mathrm{o}} \cdot \, d\vec{\ell} = -V_i^{\mathrm{o}} \int_{\vec{\rho}_0}^{\vec{\rho}_n} \nabla_{\mathrm{T}} \Phi_i^{\mathrm{o}}(\vec{\rho}) \cdot \, d\vec{\ell} = V_i^{\mathrm{o}} \Big(\Phi_i^{\mathrm{o}}(\vec{\rho}_n) - \Phi^{\mathrm{o}}(\vec{\rho}_0) \Big)$$

$$= \quad V_i^{\mathrm{o}} \, \mathrm{cost}_i^{(n)}$$

$$\vec{J}_s = -\vec{n} \times \vec{H}_i^{\mathrm{o}} = \, I_i^{\mathrm{o}} \, \vec{n} \times (\vec{u}_z \times \nabla_{\mathrm{T}} \Phi_i^{\mathrm{o}}) = \vec{u}_z \; I_i^{\mathrm{o}} \, \frac{\partial \Phi_i^{\mathrm{o}}}{\partial n}$$

$$I_i^{(n)} \, = \, I_i^{\mathrm{o}} \int_{C_n} \frac{\partial \Phi_i^{\mathrm{o}}}{\partial n} \, dc$$

$$Z_i^{(n)} \, = \eta \, \mathrm{cost}_i^{(n)} \, \left(\int_{C_n} \frac{\partial \Phi_i^{\mathrm{o}}}{\partial n} \, dc \right)^{-1}$$

impedenza caratteristica

$$
\begin{aligned}
P_i^{\mathrm{o}} &= \frac{1}{2}\,\mathrm{Re}\left(\int_S \vec{E}_i^{\mathrm{o}} \times \vec{H}_i^{\mathrm{o}*} \cdot \vec{u}_z \, ds\right) \\
&= \frac{1}{2}\,\mathrm{Re}\Big(V_i^{\mathrm{o}}\, I_i^{\mathrm{o}\,*}\Big) \int_S \vec{e}_i^{\;\mathrm{o}} \times \vec{h}_i^{\mathrm{o}} \cdot \vec{u}_z \, ds
\end{aligned}
$$

$$
\begin{aligned}
P_i^{\mathrm{o}} &= \frac{1}{2}\,\mathrm{Re}\left(\sum_{n=1}^{N} \Big(V_i^{\mathrm{o}} \,\mathrm{cost}_i^{(n)}\Big)\Big(I_i^{\mathrm{o}} \int_{C_n} \frac{\partial \Phi_i^{\mathrm{o}}}{\partial n}\, dc\Big)^{*}\right) \\
&= \frac{1}{2} \sum_{n=1}^{N} \mathrm{Re}\left(V_i^{(n)}\, I_i^{(n)\,*}\right)
\end{aligned}
$$

$$
\int_S \vec{e}_i^{\;\mathrm{o}} \times \vec{h}_j^{\mathrm{o}} \cdot \vec{u}_z \, ds \quad = \begin{cases} \displaystyle\sum_{n=1}^{N} \mathrm{cost}_i^{(n)} \int_{C_n} \frac{\partial \Phi_j^{\mathrm{o}}}{\partial n}\, dc & i = j \\ \mathbf{0} & i \neq j \end{cases}
$$

modo TEM (un solo conduttore interno)

$\nabla_T^2\,\Phi^o = 0$		equazione
$\Phi^o(\vec{\rho}) = 0$	$\vec{\rho} \in C_0$	condizione al contorno
$\Phi^o(\vec{\rho}) = 1$	$\vec{\rho} \in C_1$	
$\gamma^o = jk$		costante di propagazione
$\vec{e}^{\,o} = -\nabla_T \Phi^o$		vettori modali
$\vec{h}^{\,o} = \dfrac{Z_c}{\eta}\,\vec{u}_z \times \vec{e}^{\,o}$		
$Z_c = \eta \left(\displaystyle\int_{C_1} \frac{\partial \Phi^o}{\partial n}\, dc \right)^{-1}$		impedenza caratteristica
$\vec{E}^o = \vec{e}^{\,o}\, V$		campi modali
$\vec{H}^o = \vec{h}^{\,o}\, I$		

$$\int_S \vec{e}^{\,o} \cdot \vec{e}^{\,o}\, ds = \frac{\eta}{Z_c} \qquad \int_S \vec{h}^{\,o} \cdot \vec{h}^{\,o}\, ds = \frac{Z_c}{\eta} \qquad \int_S \vec{e}^{\,o} \times \vec{h}^{\,o} \cdot \vec{u}_z\, ds = 1$$

modo TEM del cavo coassiale

$$\Phi^{\circ} = \frac{\ln(r_e/r)}{\ln(r_e/r_i)}$$

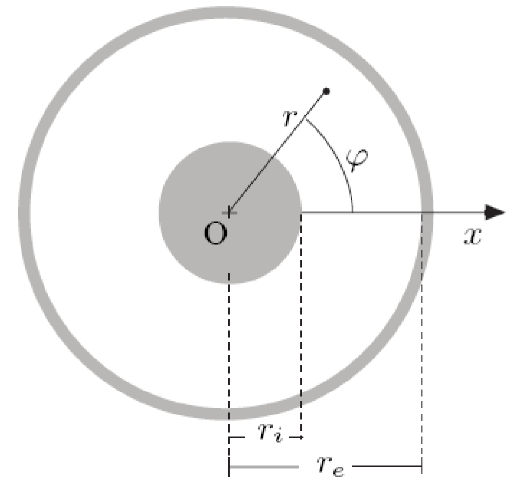
$$\vec{e}^{\,\circ} = \frac{1}{r\,\ln(r_e/r_i)}\,\vec{u}_r$$

$$\vec{h}^{\,\circ} = \frac{Z_c}{\eta}\,\vec{u}_z \times \vec{e}^{\,\circ} = \frac{1}{2\pi\, r}\,\vec{u}_\varphi$$

$$Z_c = \frac{\eta}{2\pi}\,\ln(r_e/r_i)$$

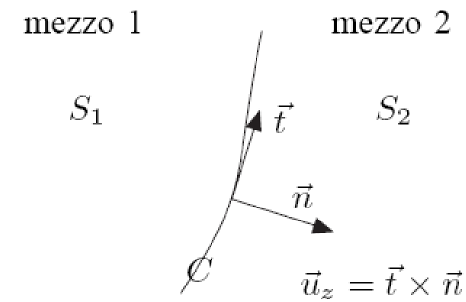
$$\vec{E}^{\circ}(r,\varphi,z) = \vec{e}^{\,\circ}(r,\varphi)\,V(z)$$

$$\vec{H}^{\circ}(r,\varphi,z) = \vec{h}^{\,\circ}(r,\varphi)\,I(z)$$



il primo modo superiore è il TE_{11} : $\lambda''_{11} \approx \pi\,(r_i + r_e)$

condizioni su un'interfaccia



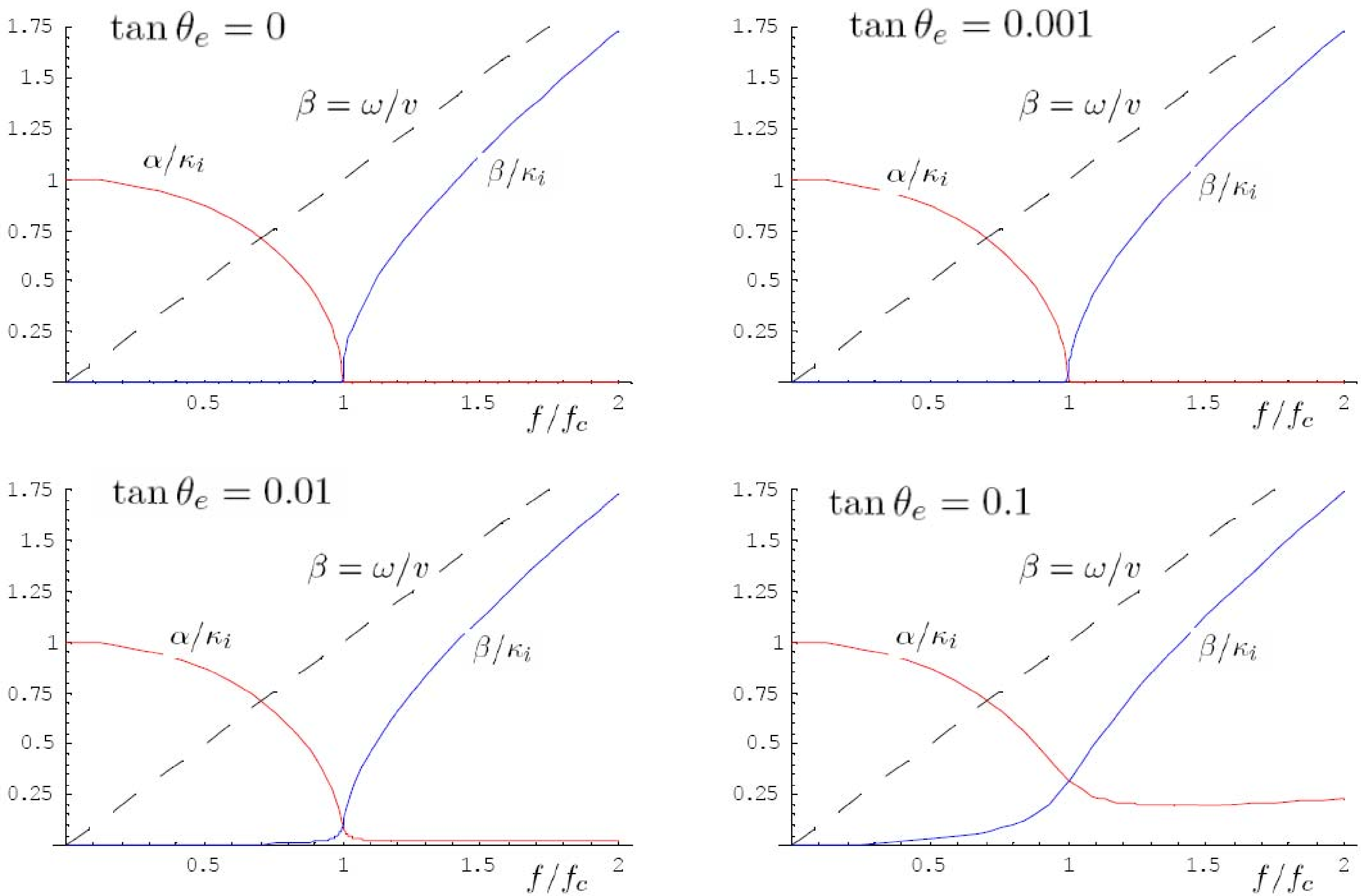
$$\nabla_{\mathrm{T}}^2 \, \Phi_1(\vec{\rho}) + \kappa_1^2 \, \Phi_1(\vec{\rho}) = 0 \qquad \vec{\rho} \in S_1$$

$$\kappa_1^2 = k_1^2 + \gamma^2$$

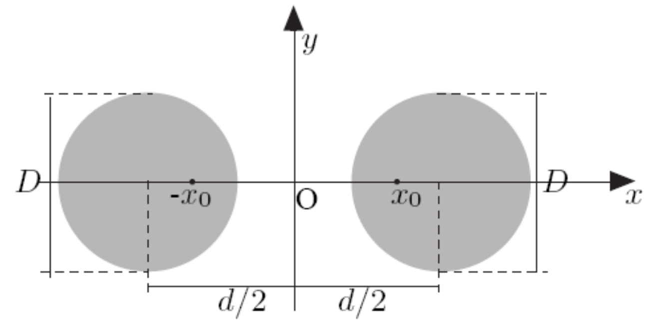
$$\nabla_{\mathrm{T}}^2 \, \Phi_2(\vec{\rho}) + \kappa_2^2 \, \Phi_2(\vec{\rho}) = 0 \qquad \vec{\rho} \in S_2$$

$$\kappa_2^2 = k_2^2 + \gamma^2$$

$$\left.\begin{aligned}
\frac{\eta_1 \, \gamma}{j \, k_1 \, Z} \, \frac{\partial \, \Phi_1'(\vec{\rho})}{\partial t} + \frac{\partial \, \Phi_1''(\vec{\rho})}{\partial n} &= \frac{\eta_2 \, \gamma}{j \, k_2 \, Z} \, \frac{\partial \, \Phi_2'(\vec{\rho})}{\partial t} + \frac{\partial \, \Phi_2''(\vec{\rho})}{\partial n} \\
\frac{\gamma \, Z}{j \, k_1 \, \eta_1} \, \frac{\partial \, \Phi_1''(\vec{\rho})}{\partial t} - \frac{\partial \, \Phi_1'(\vec{\rho})}{\partial n} &= \frac{\gamma \, Z}{j \, k_2 \, \eta_2} \, \frac{\partial \, \Phi_2''(\vec{\rho})}{\partial t} - \frac{\partial \, \Phi_2'(\vec{\rho})}{\partial n} \\
\frac{\eta_1 \, \kappa_1^2}{k_1} \, \Phi_1'(\vec{\rho}) &= \frac{\eta_2 \, \kappa_2^2}{k_2} \, \Phi_2'(\vec{\rho}) \\
\frac{\kappa_1^2}{k_1 \, \eta_1} \, \Phi_1''(\vec{\rho}) &= \frac{\kappa_2^2}{k_2 \, \eta_2} \, \Phi_2''(\vec{\rho})
\end{aligned}\right\} \qquad \vec{\rho} \in C$$


$\tan\theta_e = 0$
$\tan\theta_e = 0.001$
$\tan\theta_e = 0.01$
$\tan\theta_e = 0.1$
$\beta = \omega/v$
α/κ_i
β/κ_i
f/f_c
0.25
0.5
0.75
1
1.25
1.5
1.75
2

linea bifilare

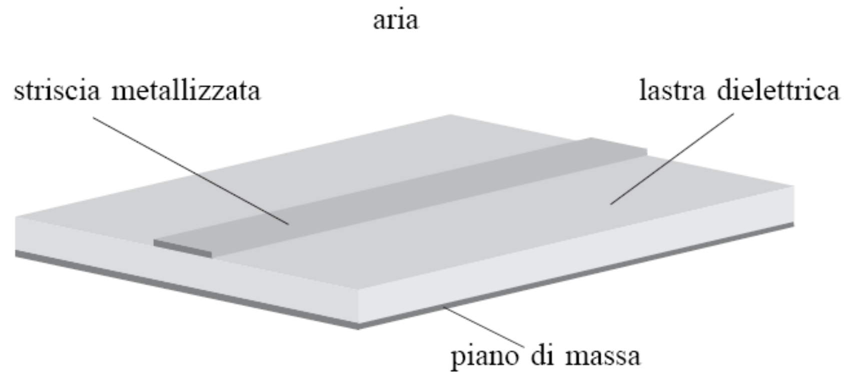


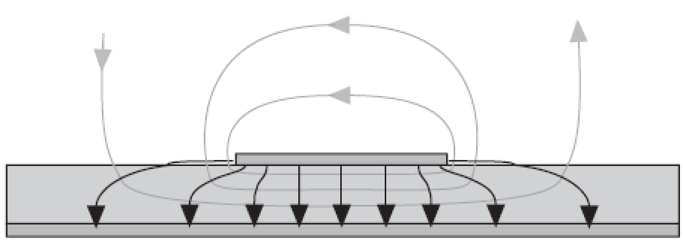
$$
\begin{aligned}
\Phi^\circ &= \frac{1}{F}\ \ln \frac{(x-x_0)^2+y^2}{(x+x_0)^2+y^2} \\
\vec{e}^{\,\circ} &= \frac{4x_0(x^2-x_0^2-y^2)\vec{u}_x + 2xy\,\vec{u}_y}{F\ ((x-x_0)^2+y^2)((x+x_0)^2+y^2)} \\
\vec{h}^{\,\circ} &= \frac{Z_c}{\eta}\ \frac{-2xy\,\vec{u}_x + 4x_0(x^2-x_0^2-y^2)\vec{u}_y}{F\ ((x-x_0)^2+y^2)((x+x_0)^2+y^2)} \\
Z_c &= \frac{\eta}{\pi}\ \mathrm{arccosh}\frac{d}{D} \\
\vec{E}^\circ &= \vec{e}^{\,\circ}\, V^\circ(z) \\
\vec{H}^\circ &= \vec{h}^{\,\circ}\, I^\circ(z)
\end{aligned}
$$

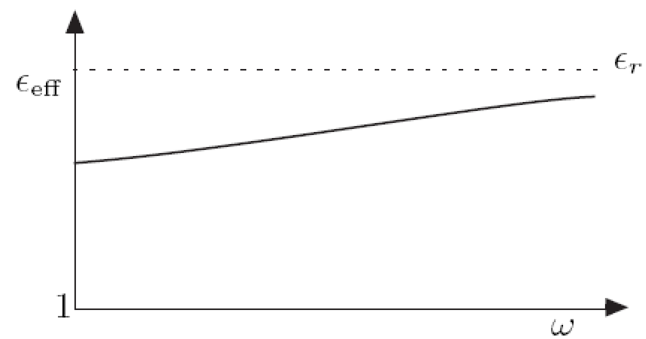
$$x_0 = \tfrac{1}{2}\sqrt{d^2 - D^2} \qquad\qquad F = \ln\,\frac{d-2x_0}{d+2x_0}$$

linea in microstriscia

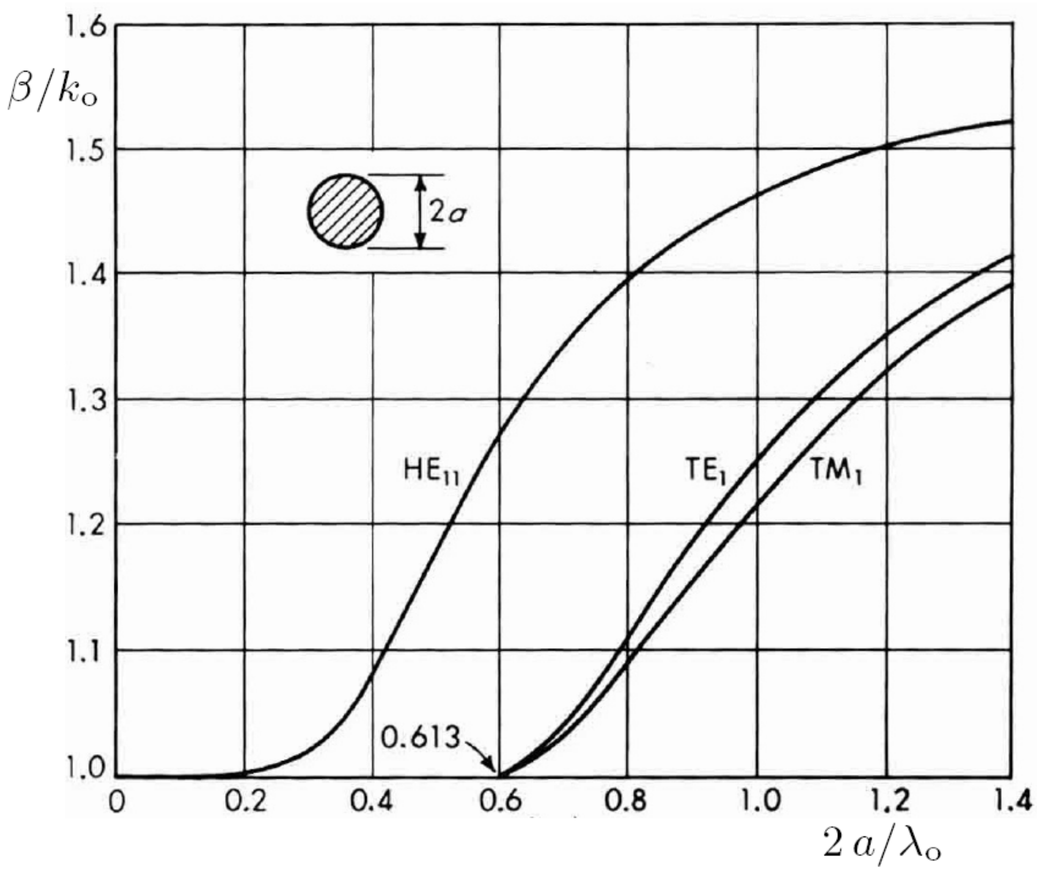
$$\epsilon_{\text{eff}} = \frac{\beta^2}{(\omega/c)^2}$$







guida dielettrica



diagrammi di dispersione dei primi modi confinati di una guida dielettrica a sezione circolare in polistirene (ε_r = 2.56).

lastra dielettrica

$$0 \leq x < d/2$$

$$\frac{d^2\,\Phi''(x)}{d\,x^2} + (\,\underbrace{(\omega/c)^2\,n^2 - \beta^2}_{k_x^2}\,)\,\Phi''(x) \;=\; 0$$

$$d/2 < x$$

$$\frac{d^2\,\Phi''(x)}{d\,x^2} + (\,\underbrace{(\omega/c)^2 - \beta^2}_{-\alpha^2}\,)\,\Phi''(x) \;=\; 0$$

$$\lim_{x \to \infty} \quad \Phi''(x) = 0$$

$$\lim_{x \to d/2^-} \frac{d\,\Phi''(x)}{d\,x} \;=\; \lim_{x \to d/2^+} \frac{d\,\Phi''(x)}{d\,x}$$

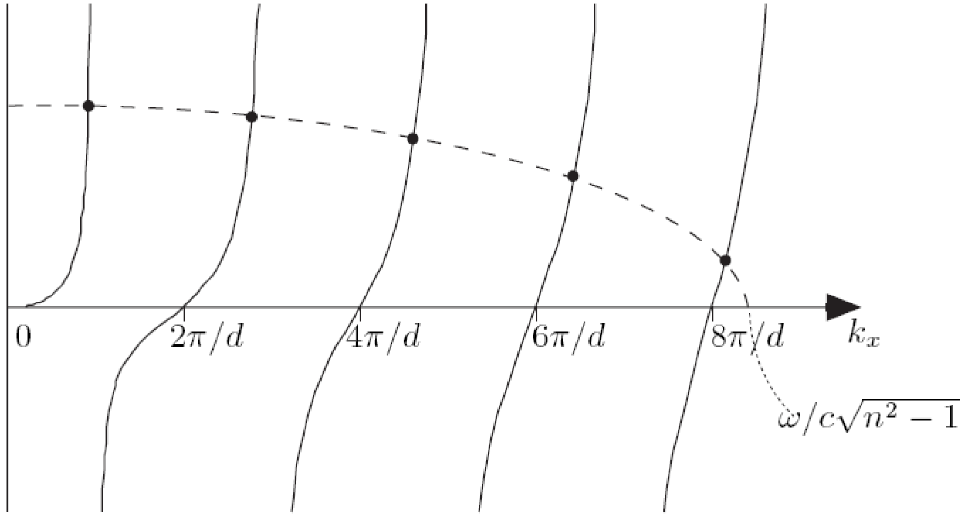
$$\lim_{x \to d/2^-} k_x^2\,\Phi''(x) \;=\; \lim_{x \to d/2^+} -\alpha^2\,\Phi''(x)$$

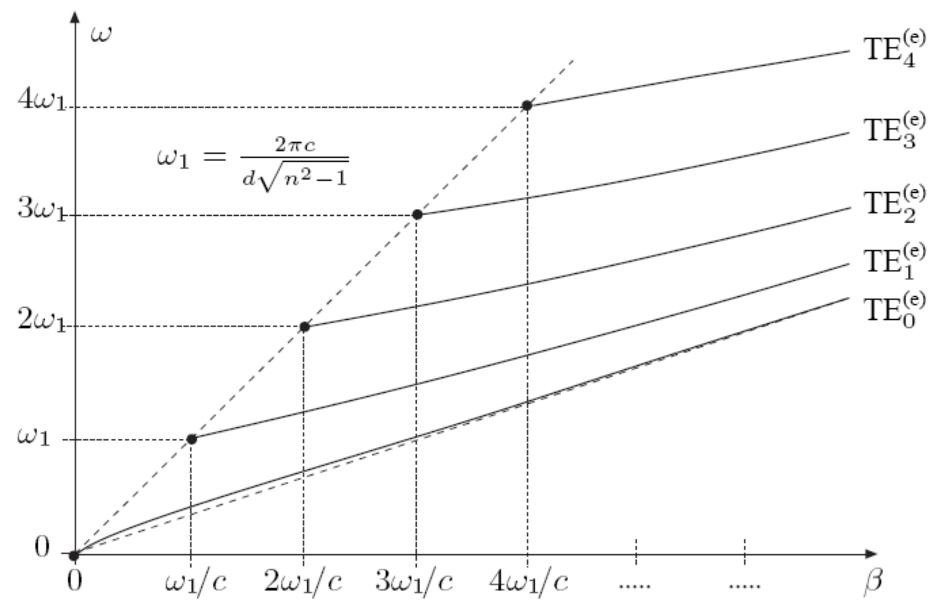
$0 \leq x < d/2$ $\qquad$ $d/2 < x$

$$\Phi''(x) \;=\; B\,\sin(k_x x) \qquad \Phi''(x) \;=\; A\,e^{-\alpha x} \qquad \mathrm{Re}(\alpha) > 0$$

$$\begin{aligned} B\,k_x \cos k_x d/2 &= -\alpha\, A e^{-\alpha d/2} \\ B\,k_x^2\, \sin k_x d/2 &= -\alpha^2\, A e^{-\alpha d/2} \end{aligned}$$

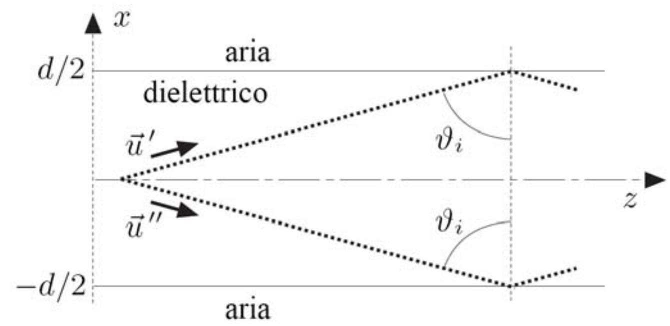
$$\boxed{k_x\ \tan k_x d/2 \;=\; \underbrace{\sqrt{(\omega/c)^2\,(n^2-1) - k_x^2}}_{\alpha}}$$


0
$2\pi/d$
$4\pi/d$
$6\pi/d$
$8\pi/d$
k_x
$\omega/c\sqrt{n^2-1}$


ω
$4\omega_1$
$3\omega_1$
$2\omega_1$
ω_1
0
$\omega_1 = \frac{2\pi c}{d\sqrt{n^2-1}}$
$TE_4^{(e)}$
$TE_3^{(e)}$
$TE_2^{(e)}$
$TE_1^{(e)}$
$TE_0^{(e)}$
0
ω_1/c
$2\omega_1/c$
$3\omega_1/c$
$4\omega_1/c$
.....
.....
β

$$\vec{E} = \underbrace{\vec{u}_y \frac{A\,k_x}{2}\, e^{-jk_\circ n\,\vec{u}\,'\cdot\vec{r}}}_{\vec{E}\,'} + \underbrace{\vec{u}_y \frac{A\,k_x}{2}\, e^{-jk_\circ n\,\vec{u}\,''\cdot\vec{r}}}_{\vec{E}\,''}$$

$$\vec{H} = \underbrace{\frac{n}{\eta_\text{o}}\,\vec{u}\,'\times\vec{E}\,'}_{\vec{H}\,'} + \underbrace{\frac{n}{\eta_\text{o}}\,\vec{u}\,''\times\vec{E}\,''}_{\vec{H}\,''}$$



$$\vec{u}\,' = \frac{k_x\,\vec{u}_x + \beta\,\vec{u}_z}{k_\text{o}\,n}$$

$$\vec{u}\,'' = \frac{-k_x\,\vec{u}_x + \beta\,\vec{u}_z}{k_\text{o}\,n}$$

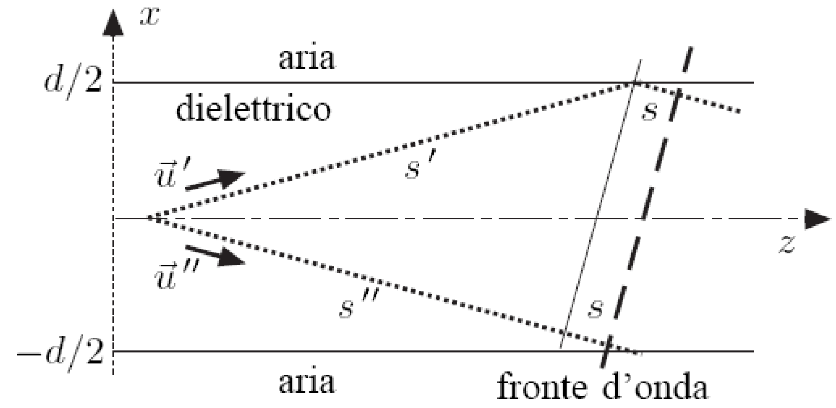
$$\tan\vartheta_i = \frac{\beta}{k_x} = \frac{\sqrt{1+n^2\,\tan^2(k_x d/2)}}{\sqrt{n^2-1}} > \frac{1}{\sqrt{n^2-1}} = \tan\vartheta_L$$

$$\beta\sqrt{n^2-1} = \sqrt{k_x^2+n^2\,\alpha^2} = k_x\sqrt{1+n^2\,\tan^2 k_x d/2}$$

$$\sin\vartheta_L = 1/n$$

$$\sin\vartheta_i = \beta/(k_o\, n)$$

$$\cos\vartheta_i = k_x/(k_o\, n)$$



$$-k_o n\, s' + \arg(\Gamma_\perp) - k_o n\, s = -k_o n\,(s'' + s)$$

$$\arg(\Gamma_\perp) = 2\arctan\left(\sqrt{\sin^2\vartheta_i - \sin^2\vartheta_L}\;/\cos\vartheta_i\right)$$

$$s' = d/2\sqrt{1+\tan^2\vartheta_i} = \frac{k_o n\, d}{2\,k_x}$$

$$s'' = s'\cos\left(2(\pi/2-\vartheta_i)\right) = -s'\cos(2\,\vartheta_i) = s'\left(1 - 2\cos^2\vartheta_i\right)$$

attenuazione nelle guide reali

$$\alpha = \frac{1}{2P}\left(\frac{dP_c}{dz} + \frac{dP_d}{dz}\right)$$

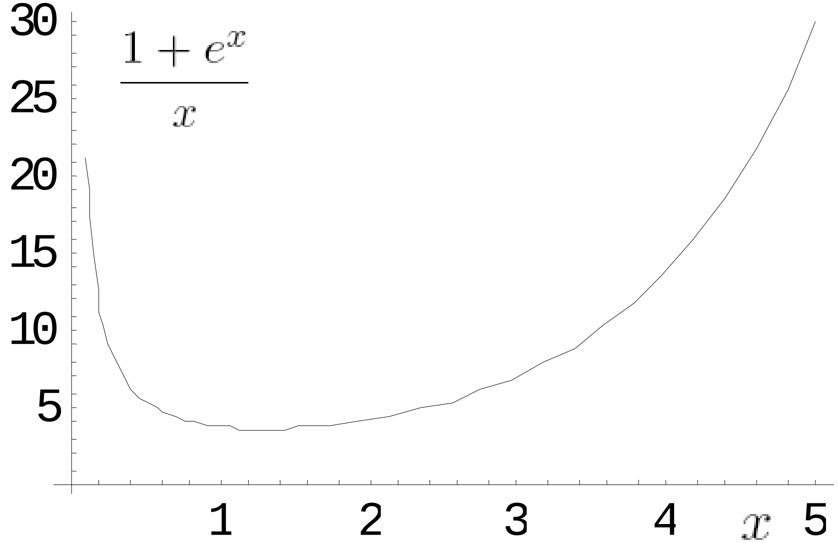
$$= \frac{\int_C R_s \mid \vec{H} \mid^2 dc + \omega \int_S (\epsilon \, \tan\theta_e \mid E \mid^2 + \mu \, \tan\theta_m \mid H \mid^2)\, ds}{2\int_S \mathrm{Re}\left(\vec{E} \times \vec{H}^*\right) \cdot \vec{u}_z \, ds}$$

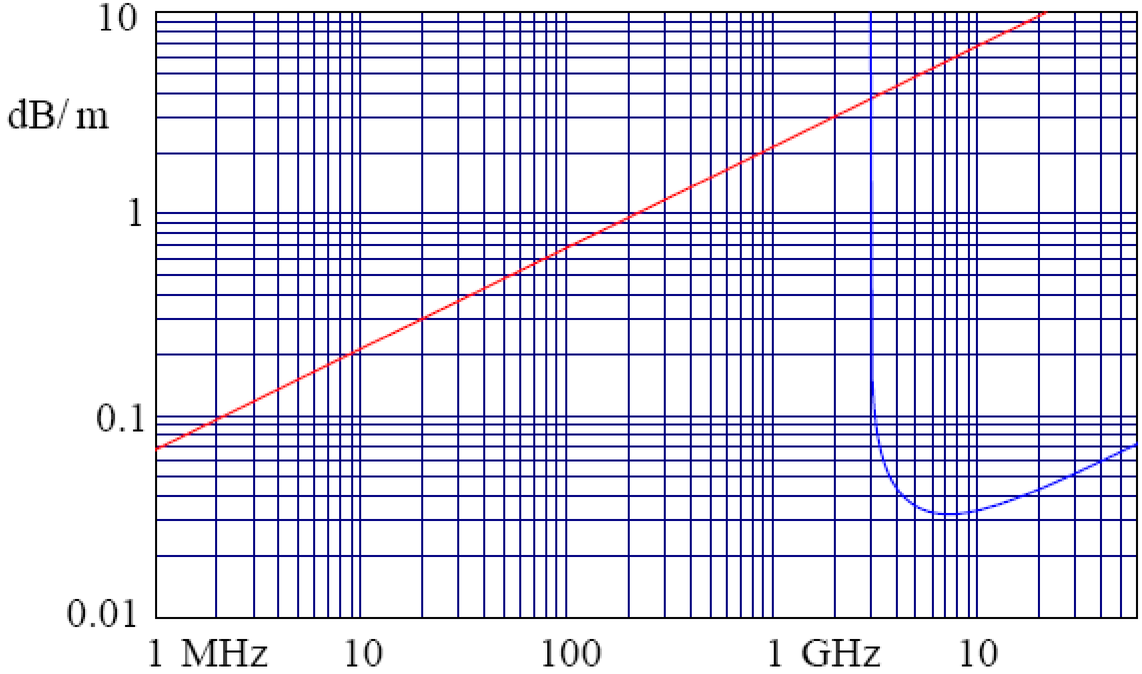
$$dP_c = \frac{1}{2}\, dz \int_c R_s \mid \vec{J}_s \mid^2 dc$$

$$dP_d = \frac{1}{2}\, \omega\, dz \int_S \left(\epsilon \, \tan\theta_e \mid E \mid^2 + \mu \, \tan\theta_m \mid H \mid^2\right) ds$$

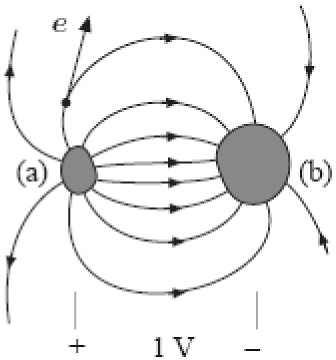
$$2\,\alpha_c\,r_e\,\eta/R_s = \frac{1+e^x}{x}$$

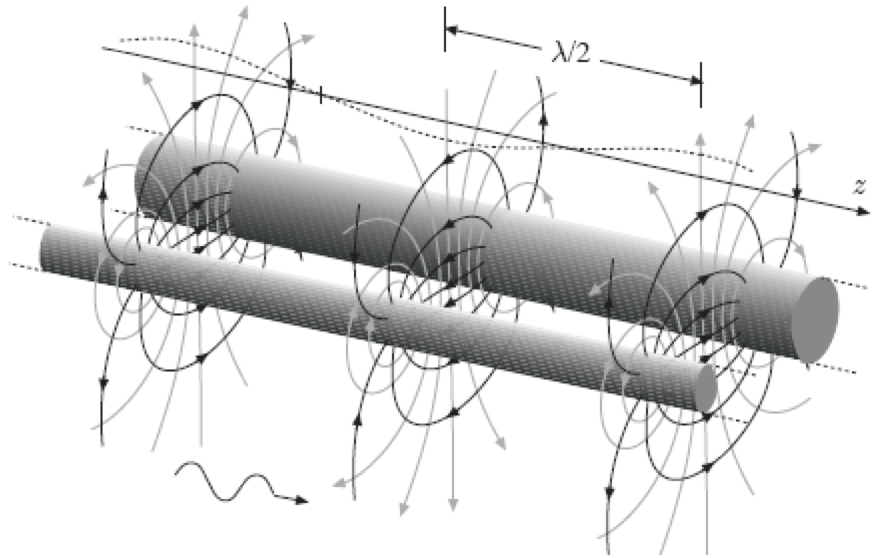
$$x = 2\,\pi Z_c/\eta = \ln\left(r_e/r_i\right)$$

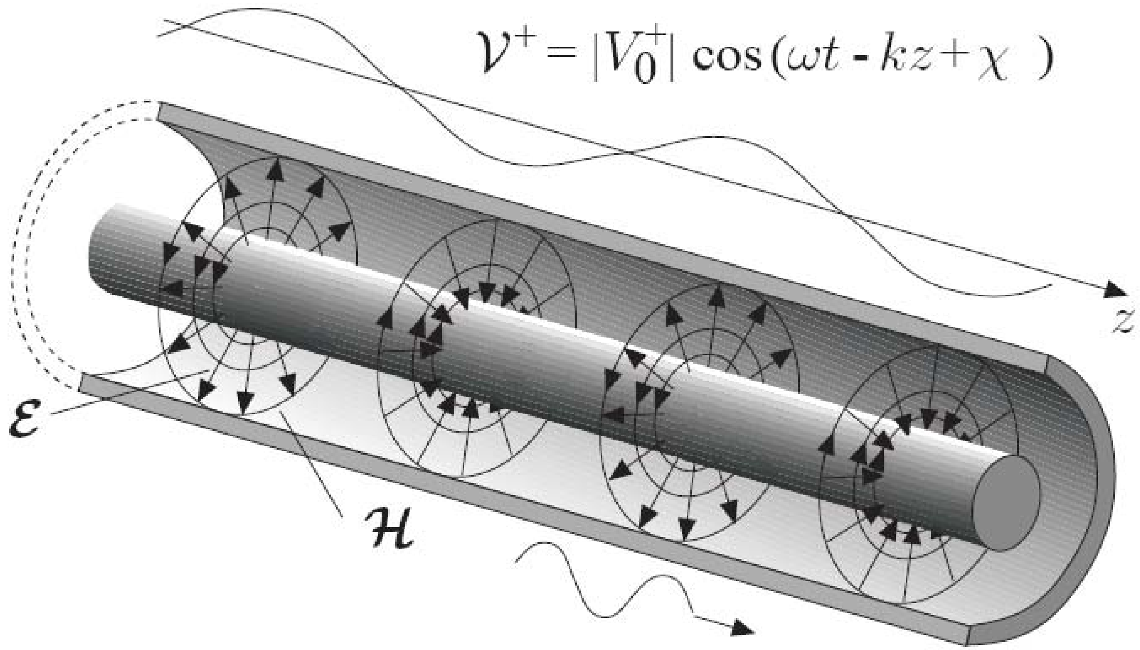


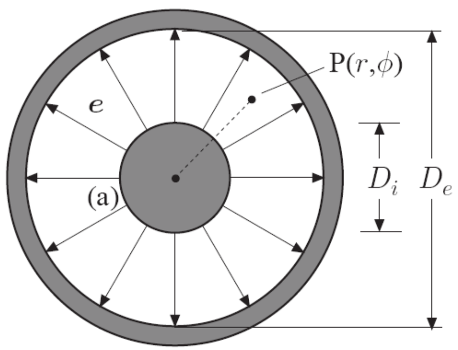


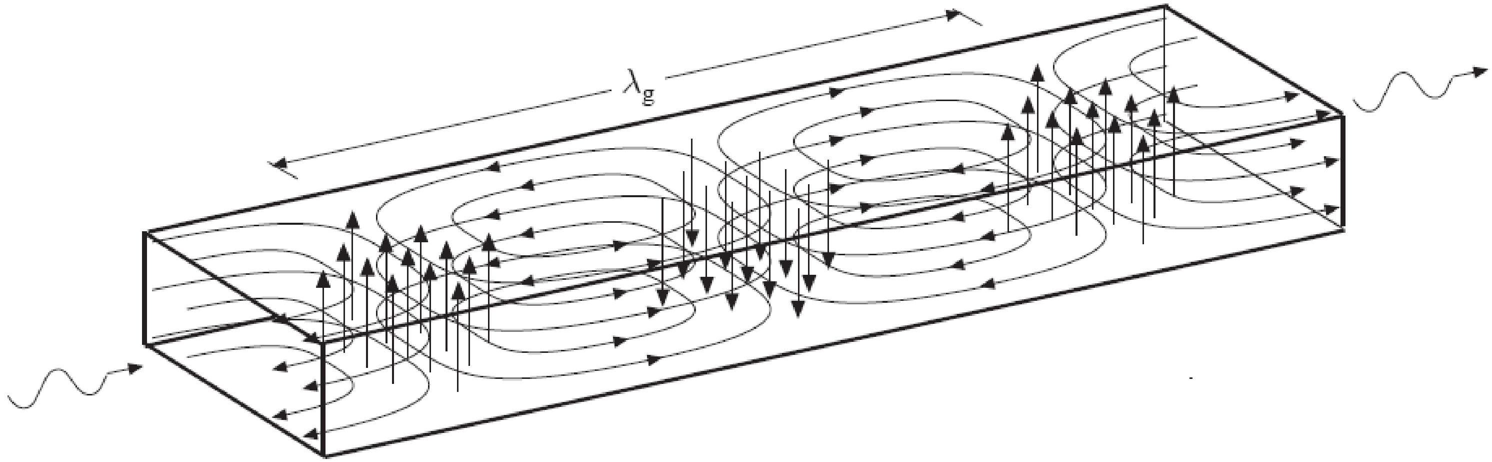
Andamento dell'attenuazione del modo TEM di un tipico cavo coassiale e del modo TE_{10} di una guida rettangolare in aria


e
(a)
(b)
+
1 V
–


λ/2
z


$\mathcal{V}^+ = |V_0^+| \cos(\omega t - kz + \chi\ )$
z
$\mathcal{E}$
$\mathcal{H}$


P(r,ϕ)
e
(a)
D_i
D_e



λ_g