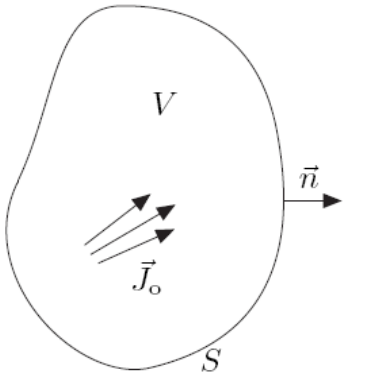
teorema di unicità

$$\begin{aligned} \nabla \times \vec{E}^a &= -j\omega\mu\, \vec{H}^a \\ \nabla \times \vec{H}^a &= j\omega\epsilon\, \vec{E}^a + \vec{J}_o \end{aligned} \qquad \begin{aligned} \nabla \times \vec{E}^b &= -j\omega\mu\, \vec{H}^b \\ \nabla \times \vec{H}^b &= j\omega\epsilon\, \vec{E}^b + \vec{J}_o \end{aligned}$$



$$\delta\vec{E} = \vec{E}^a - \vec{E}^b \qquad \delta\vec{H} = \vec{H}^a - \vec{H}^b$$

$$\begin{aligned} \nabla \times\ \delta\vec{E} &= -j\omega\mu\, \delta\vec{H} \\ \nabla \times\ \delta\vec{H} &= j\omega\epsilon\, \delta\vec{E} \end{aligned}$$

$$0 = \underbrace{\frac{1}{2}\,\omega \int_V \left(\mu_o\mu''\,|\,\delta\vec{H}\,|^2 + \epsilon_o\epsilon''\,|\,\delta\vec{E}\,|^2\right)\, dV}_{\delta P_{\text{diss}}} + \underbrace{\frac{1}{2}\,\text{Re}\left(\int_S \delta\vec{E} \times \delta\vec{H}^* \cdot \vec{n}\, dS\right)}_{\delta P_{\text{out}}}$$

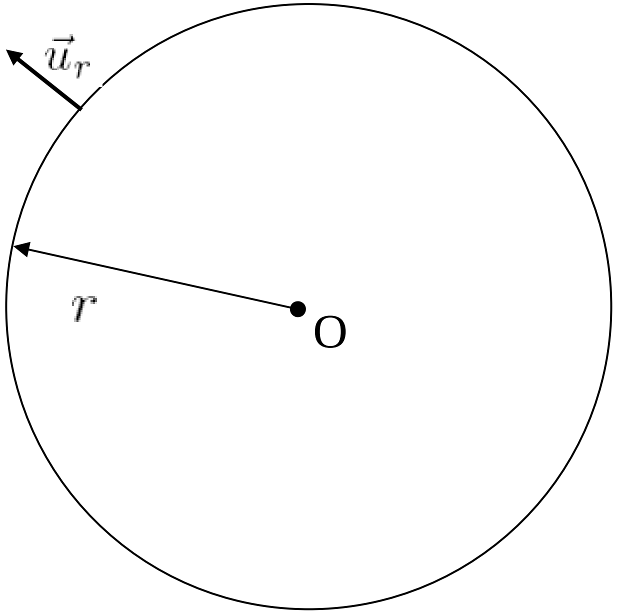
$$\vec{n} \times \vec{E} - \mathcal{Z}\, \vec{H}_{\mathrm{T}} \;=\; \vec{V}_{\mathrm{T}} \qquad \text{in } S_1 \qquad\qquad \mathrm{Re}(\mathcal{Z}) > 0$$

$$\vec{H} \times \vec{n} - \mathcal{Y}\, \vec{E}_{\mathrm{T}} \;=\; \vec{W}_{\mathrm{T}} \qquad \text{in } S_2 \qquad\qquad \mathrm{Re}(\mathcal{Y}) > 0$$

$$\delta P_{\mathrm{out}} \;=\; \tfrac{1}{2}\,\mathrm{Re}\,(\mathcal{Z}) \int_{S_1} |\, \delta\vec{H}_{\mathrm{T}}\, |^2\, dS + \tfrac{1}{2}\,\mathrm{Re}\,(\mathcal{Y}) \int_{S_2} |\, \delta\vec{E}_{\mathrm{T}}\, |^2\, dS \;\geq\; 0$$

- *l'assegnazione del campo elettrico tangenziale*
  $\vec{n} \times \vec{E} = \vec{V}_{\mathrm{T}}$
- *l'assegnazione del campo magnetico tangenziale*
  $\vec{H} \times \vec{n} = \vec{W}_{\mathrm{T}}$
- *la condizione di parete elettrica*
  $\vec{n} \times \vec{E} = 0$
- *la condizione di parete magnetica*
  $\vec{n} \times \vec{H} = 0$
- *la condizione d'impedenza*
  $\vec{n} \times \vec{E} = \mathcal{Z}\, \vec{H}_{\mathrm{T}}$
- *la condizione di ammettenza*
  $\vec{H} \times \vec{n} = \mathcal{Y}\, \vec{E}_{\mathrm{T}}$
- *la condizione di radiazione*

Condizione di radiazione



$$\left.\begin{aligned} \lim_{r\to\infty} r^\nu \vec{E} &= 0 \\ \lim_{r\to\infty} r^\nu \vec{H} &= 0 \end{aligned}\right\} \quad \forall\, \nu < 1$$

$$\lim_{r\to\infty} r\left(\vec{u}_r \times \vec{E} - \eta_\infty \vec{H}\right) = 0$$

$$\lim_{r\to\infty} r\left(\vec{H} \times \vec{u}_r - \vec{E}\,/\,\eta_\infty\right) = 0$$

radiazione

$$\vec{A} = \mu \int_V \frac{e^{-jkR}}{4\,\pi\,R}\; \vec{J}_{\rm o}(\vec{r}\,')\, dV\,'$$

$$\begin{aligned}
\vec{E} &= -j\omega\left(\vec{A} + \frac{1}{k^2}\nabla\nabla\cdot\vec{A}\right) \\
&= -j\eta k \int_V \frac{e^{-jkR}}{4\,\pi\,R}\left[\left(1 - \frac{j}{kR} - \frac{1}{k^2R^2}\right)\vec{J}_{\rm o}(\vec{r}\,')\right. \\
&\qquad\qquad \left. -\left(1 - \frac{3j}{kR} - \frac{3}{k^2R^2}\right)\vec{u}_R\left(\vec{u}_R\cdot\vec{J}_{\rm o}(\vec{r}\,')\right)\right] dV\,'
\end{aligned}$$

$$\begin{aligned}
\vec{H} &= \frac{\nabla\times\vec{A}}{\mu} \\
&= -j\,k \int_V \frac{e^{-jkR}}{4\,\pi\,R}\left(1 - \frac{j}{kR}\right)\,\vec{u}_R\,\times\vec{J}_{\rm o}(\vec{r}\,')\,dV\,'
\end{aligned}$$

$$\vec{u}_R = \frac{\vec{R}}{R}$$

zona di radiazione

$$r \gg D \qquad \Rightarrow \qquad \frac{1}{R} \approx \frac{1}{r} \qquad \vec{u}_R \approx \vec{u}_r$$

$$r > \frac{2D^2}{\lambda} \qquad \Rightarrow \qquad e^{-jkR} \approx e^{-jkr}\, e^{j\vec{u}_r\cdot\vec{r}\,'}$$

$$r \gg \lambda \qquad \Rightarrow \qquad \frac{1}{k^2\, r^2} \ll \frac{1}{k\, r} \ll 1$$

$$\vec{A} \approx \mu\, \frac{e^{-jkr}}{4\,\pi\, r}\, \underbrace{\int_V e^{jk\vec{u}_r\cdot\vec{r}\,'}\; \vec{J}_{\rm o}(\vec{r}\,')\, dV\,'}_{\vec{N}(\vartheta,\varphi)}$$

$$\vec{E} \approx -j\eta k\, \frac{e^{-jkr}}{4\,\pi\, r}\, \left(\vec{N} - \vec{u}_r\big(\vec{u}_r\cdot\vec{N}\big)\right)$$

$$\vec{H} \approx \frac{\vec{u}_r \times \vec{E}}{\eta}$$

zona dei campi reattivi

$$r \gg D \qquad \Rightarrow \qquad \frac{1}{R} \approx \frac{1}{r} \qquad \vec{u}_R \approx \vec{u}_r$$

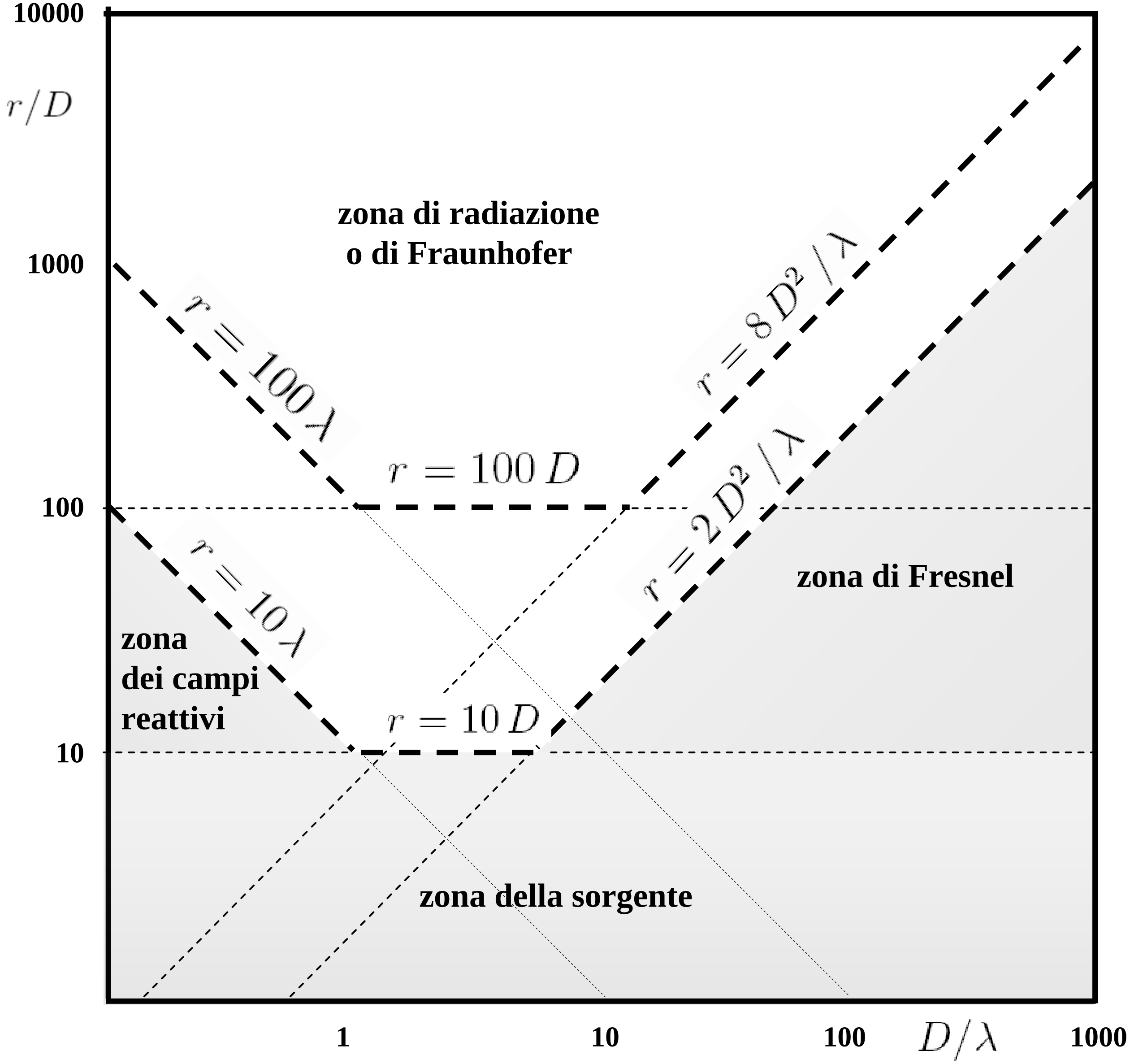
$$r > \frac{2D^2}{\lambda} \qquad \Rightarrow \qquad e^{-jkR} \approx e^{-jkr}\, e^{j\vec{u}_r \cdot \vec{r}\,'}$$

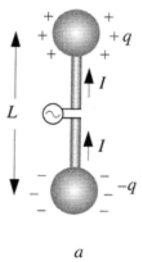
$$r \ll \lambda \qquad \Rightarrow \qquad 1 \ll \frac{1}{k\,r} \ll \frac{1}{k^2\,r^2}$$

$$\vec{A} \approx \mu\, \frac{e^{-jkr}}{4\,\pi\,r} \underbrace{\int_V e^{jk\vec{u}_r \cdot \vec{r}\,'}\, \vec{J}_{\rm o}(\vec{r}\,')\, dV\,'}_{\vec{N}(\vartheta,\varphi)}$$

$$\vec{E} \;\approx\; -j\eta k\, \frac{e^{-jkr}}{4\,\pi\,r} \left[\left(\cancel{1} - \cancel{\frac{j}{kr}} - \frac{1}{k^2 r^2} \right) \vec{N} - \left(\cancel{1} - \cancel{\frac{3j}{kr}} - \frac{3}{k^2 r^2} \right) \vec{u}_r \left(\vec{u}_r \cdot \vec{N} \right) \right]$$

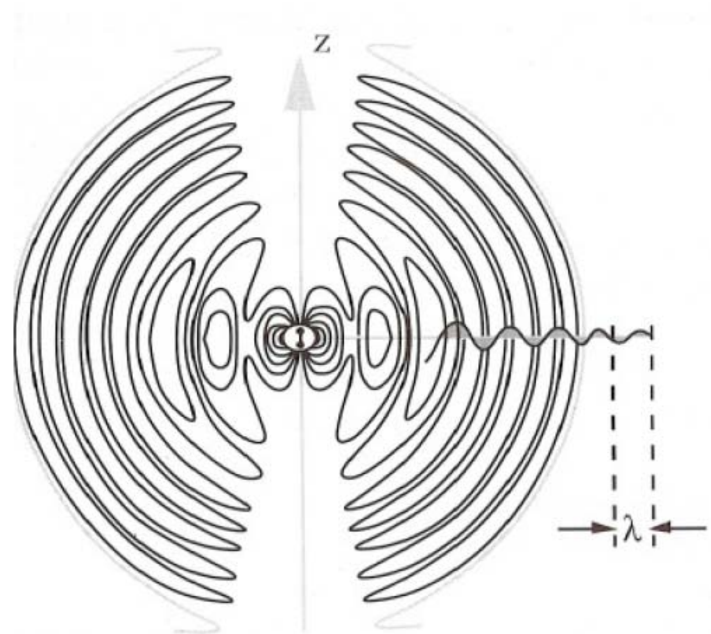
$$\vec{H} \;\approx\; -j\,k\, \frac{e^{-jkr}}{4\,\pi\,r} \left(\cancel{1} - \frac{j}{kr} \right) \vec{u}_r \times \vec{N}$$


10000
1000
100
10
r/D
1
10
100
1000
D/λ
zona di radiazione
o di Fraunhofer
$r = 100\lambda$
$r = 8D^2/\lambda$
$r = 100\,D$
$r = 2D^2/\lambda$
$r = 10\lambda$
zona di Fresnel
zona
dei campi
reattivi
$r = 10\,D$
zona della sorgente



campo di radiazione del dipolo hertziano

$(r \gg \lambda)$



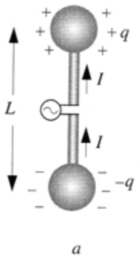
$$\vec{N} = I\, L\, \vec{u}_z$$

$$\vec{E} = j\eta_o\, I\, L\, \frac{e^{-j\,k\,r}}{2\,\lambda_o\, r}\, \sin\vartheta\; \vec{u}_\vartheta$$

$$\vec{H} = j\, I\, L\, \frac{e^{-j\,k\,r}}{2\,\lambda_o\, r}\, \sin\vartheta\; \vec{u}_\varphi$$

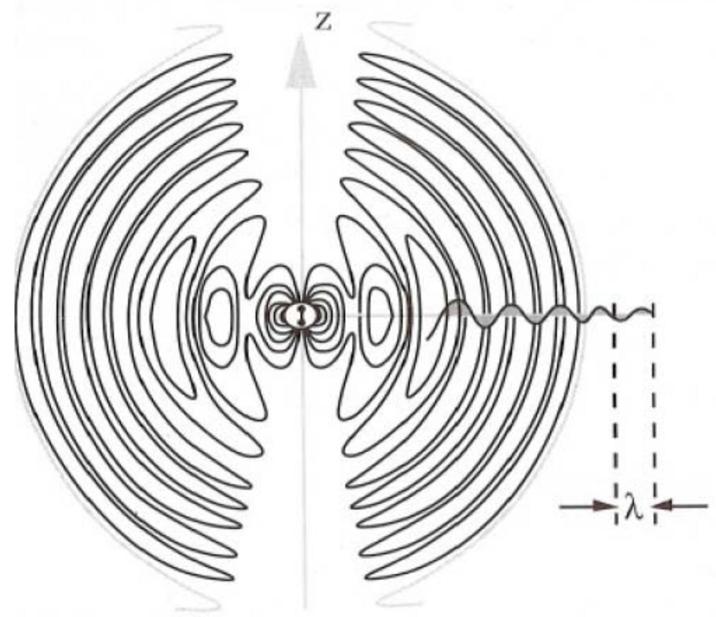
$$K = \frac{\eta_o}{8} \left(\frac{L}{\lambda_o}\right)^2 |\, I|^2\; \sin^2\vartheta$$

$$P_{\mathrm{irr}} = \frac{\eta_o\, \pi}{3} \left(\frac{L}{\lambda_o}\right)^2 |\, I|^2$$



campo reattivo del dipolo hertziano

$$(r \ll \lambda ,\ r \gg L)$$



$$\vec{E} \approx \frac{q\,L}{4\,\pi\,\epsilon_o\,r^3}\,(2\,\cos\vartheta\,\vec{u}_r + \sin\vartheta\,\vec{u}_\vartheta)$$

$$= -\nabla\,\frac{q\,L}{4\,\pi\,\epsilon_o\,r^2}\,\cos\vartheta$$

$$\vec{H} \approx \frac{I\,L}{4\,\pi\,r^2}\,\sin\vartheta\,\vec{u}_\varphi$$

$$= \frac{I\,L}{4\,\pi}\,\frac{\vec{u}_z \times \vec{u}_r}{r^2} \qquad \text{(Biot-Savart)}$$

$$I = j\omega\,q$$

$$\frac{\eta_o\,\lambda_o}{2\,\pi} = \frac{1}{\epsilon_o\,\omega}$$

spira di piccole dimensioni

$$\vec{N} = j\,k\,S\,I \sin\vartheta\,\vec{u}_\varphi$$

$$\vec{E} = \eta_o\,I\,\frac{\pi\,S}{\lambda_o^2}\;\frac{e^{-jkr}}{r}\;\sin\vartheta\,\vec{u}_\varphi$$

$$\vec{H} = -I\;\frac{\pi\,S}{\lambda_o^2}\;\frac{e^{-jkr}}{r}\;\sin\vartheta\,\vec{u}_\vartheta$$

S = area della spira
(πa^2 spira circolare)

$$K = \frac{\eta_o\,|\,I\,|^2}{2}\left(\frac{\pi\,S}{\lambda_o^2}\right)^2 \sin^2\vartheta$$

$$P_{irr} = \frac{\eta_o\,\pi\,|\,I\,|^2}{3}\left(\frac{2\pi\,S}{\lambda_o^2}\right)^2$$

dipoli di lunghezza confrontabile con la lunghezza d'onda

$$I(z) \;=\; I_o \, \frac{\sin k(L - |\,z\,|)}{\sin(kL)}$$

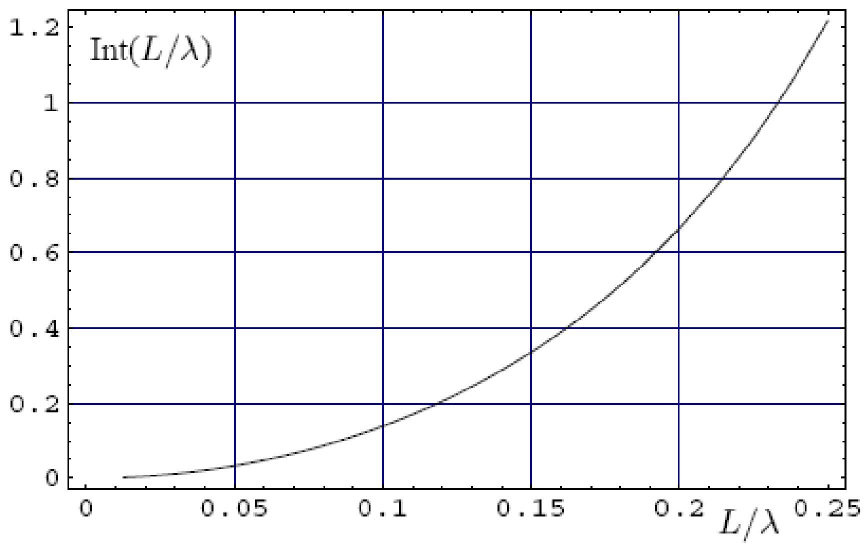
$$\vec{N} \;=\; 2\,\vec{u}_z\, I_o \, \frac{\cos(kL\cos\vartheta) - \cos(kL)}{k\,\sin(kL)\;\sin^2\vartheta}$$

$$\vec{E} \;=\; j\,\eta\, I_o\, \frac{e^{-jkr}}{2\,\pi\,r}\; \frac{\cos(kL\;\cos\vartheta) - \cos(kL)}{\sin(kL)\;\sin\vartheta}\;\vec{u}_\vartheta$$

$$\vec{H} \;=\; j\, I_o\, \frac{e^{-jkr}}{2\,\pi\,r}\; \frac{\cos(kL\;\cos\vartheta) - \cos(kL)}{\sin(kL)\;\sin\vartheta}\;\vec{u}_\varphi$$

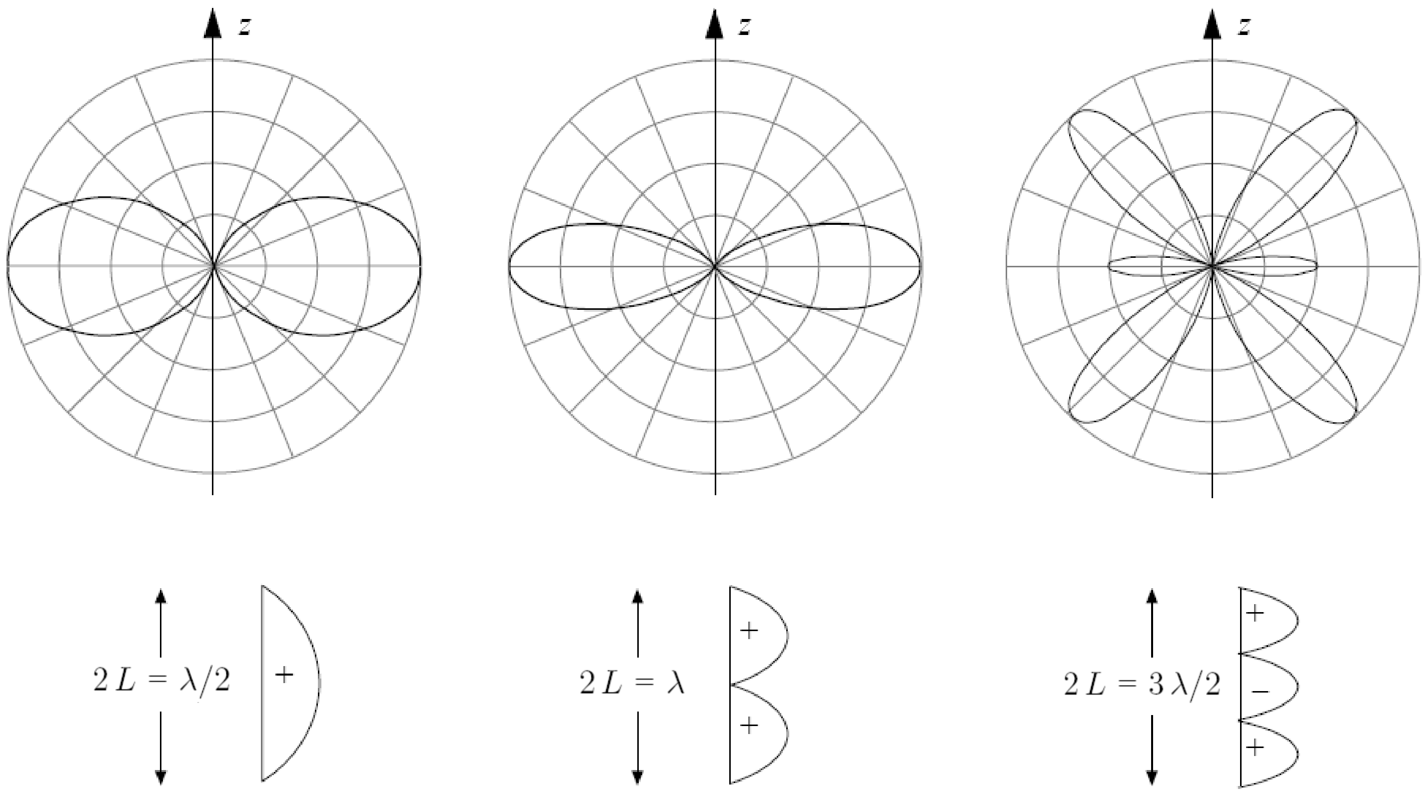
$$K \;=\; \frac{\eta}{8\,\pi^2}\,|\,I_o\,|^2\,\left(\frac{\cos(kL\;\cos\vartheta) - \cos(kL)}{\sin(kL)\;\sin\vartheta}\right)^2$$

$$\int_0^{2\pi}\int_0^{\pi}\left(\frac{\cos(kL\ \cos\vartheta)-\cos(kL)}{\sin(kL)\ \sin\vartheta}\right)^2 \sin\vartheta\, d\vartheta\, d\varphi = 2\pi\,\mathrm{Int}(L/\lambda)$$



$$P_{\mathrm{irr}} = \frac{\eta}{4\pi}\,|\,I_{\mathrm{o}}\,|^2\,\mathrm{Int}(L/\lambda)$$

$$D = \frac{2}{\mathrm{Int}(L/\lambda)}\left(\frac{\cos(kL\ \cos\vartheta)-\cos(kL)}{\sin(kL)\ \sin\vartheta}\right)^2$$


z
z
z
$2L = \lambda/2$
+
$2L = \lambda$
+
+
$2L = 3\lambda/2$
+
−
+

equazioni di Maxwell generalizzate

$$\nabla \times \vec{E} = -j\omega\mu \vec{H} - \vec{M}_o$$

$$\nabla \times \vec{H} = j\omega\epsilon\vec{E} + \vec{J}_o$$

dualità

$$\vec{E} \rightarrow \vec{H}$$

$$\vec{H} \rightarrow -\vec{E}$$

$$\vec{J}_o \rightarrow \vec{M}_o$$

$$\vec{M}_o \rightarrow -\vec{J}_o$$

$$\varepsilon \rightleftarrows \mu$$

sorgenti elettriche	sorgenti magnetiche
$\nabla \times \vec{E} = -j\omega\mu\vec{H}$	$\nabla \times \vec{E} = -j\omega\mu\vec{H} - \vec{M}_o$
$\nabla \times \vec{H} = j\omega\epsilon\vec{E} + \vec{J}_o$	$\nabla \times \vec{H} = j\omega\epsilon\vec{E}$
$\vec{E} = -j\omega\left(\vec{A} + \frac{1}{k^2}\nabla\nabla \cdot \vec{A}\right)$	$\vec{H} = -j\omega\left(\vec{F} + \frac{1}{k^2}\nabla\nabla \cdot \vec{F}\right)$
$\vec{H} = \frac{1}{\mu}\,\nabla \times \vec{A}$	$\vec{E} = -\frac{1}{\epsilon}\,\nabla \times \vec{F}$
$\vec{A} = \mu\int_V \frac{e^{-jkR}}{4\,\pi\,R}\,\vec{J}_o(\vec{r}')\,dV'$	$\vec{F} = \epsilon\int_V \frac{e^{-jkR}}{4\,\pi\,R}\,\vec{M}_o(\vec{r}')\,dV'$

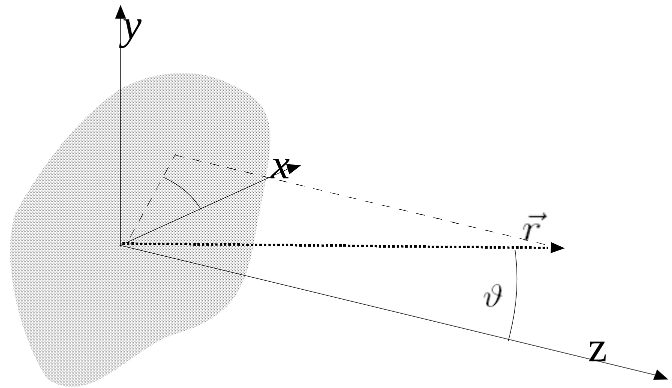
nell'approssimazione di campo di radiazione

sorgenti elettriche	sorgenti magnetiche
$\vec{A} = \mu \frac{e^{-jkr}}{4\pi r} \vec{N}$	$\vec{F} = \epsilon \frac{e^{-jkr}}{4\pi r} \vec{L}$
$\vec{N} = \int_V e^{jk\vec{u}_r\cdot\vec{r}'} \vec{J}_o(\vec{r}')\, dV'$	$\vec{L} = \int_V e^{jk\vec{u}_r\cdot\vec{r}'} \vec{M}_o(\vec{r}')\, dV'$
$\vec{E} = -j\eta \frac{e^{-jkr}}{2\lambda r} \left(\vec{N} - \vec{u}_r\left(\vec{u}_r \cdot \vec{N}\right)\right)$	$\vec{H} = -\frac{j}{\eta} \frac{e^{-jkr}}{2\lambda r} \left(\vec{L} - \vec{u}_r\left(\vec{u}_r \cdot \vec{L}\right)\right)$
$\vec{H} = -j \frac{e^{-jkr}}{2\lambda r} \vec{u}_r \times \vec{N}$	$\vec{E} = j \frac{e^{-jkr}}{2\lambda r} \vec{u}_r \times \vec{L}$
$K = \frac{\eta}{8\lambda^2} \left(\lvert \vec{N} \rvert^2 - \lvert \vec{u}_r \cdot \vec{N} \rvert^2\right)$	$K = \frac{1}{8\eta\lambda^2} \left(\lvert \vec{L} \rvert^2 - \lvert \vec{u}_r \cdot \vec{L} \rvert^2\right)$

$$\vec{H} = \frac{\vec{u}_r \times \vec{E}}{\eta} \qquad \vec{E} = \eta\, \vec{H} \times \vec{u}_r$$

$$K = \frac{\lvert \vec{E} \rvert^2}{2\eta} r^2 = \frac{\eta \lvert \vec{H} \rvert^2}{2} r^2$$

radiazione da aperture



$$\vec{L} = 2\int_{\text{apertura}} e^{j\,k\,\vec{u}_r\cdot\vec{r}\,'} M_s(\vec{r}\,') \, ds'$$

$$= 2\int_{\text{apertura}} e^{j\,k\,(u\,x'+v\,y')} M_s(x',y') \, dx'\, dy'$$

$$\vec{E} = j\,\frac{e^{-jkr}}{2\,\lambda\,r}\,\vec{u}_r\times\vec{L}$$

$$\vec{H} = -\frac{j}{\eta}\,\frac{e^{-jkr}}{2\,\lambda\,r}\,\left(\vec{L}-\vec{u}_r(\,\vec{u}_r\cdot\vec{L}\,)\right)$$

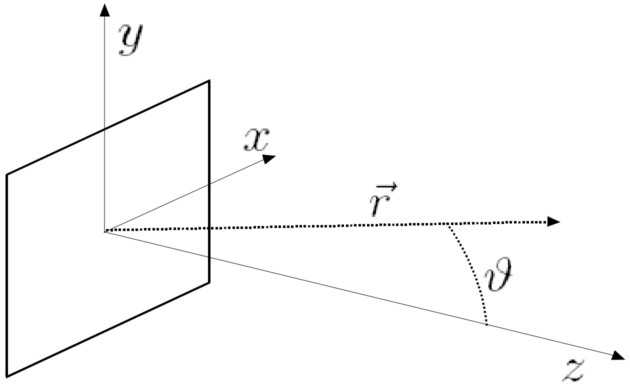
$$K = \frac{1}{8\,\eta\,\lambda^2}\,\left(|\,\vec{L}\,|^2-|\vec{u}_r\cdot\vec{L}\,|^2\right)$$

$$u = \vec{u}_r\cdot\vec{u}_x = \sin\vartheta\,\cos\varphi$$

$$v = \vec{u}_r\cdot\vec{u}_y = \sin\vartheta\,\sin\varphi$$

$$\vec{M}_s = \vec{E}\times\vec{u}_z$$

Apertura rettangolare illuminata uniformemente $\vec{E}_{\rm o} = E_{\rm o}\vec{u}_x$



$$\begin{aligned}
\vec{L} &= 2\int_{\rm apertura} e^{j\,k\,\vec{u}_r\cdot\vec{r}\,'} M_s(\vec{r}\,')\,ds' \\
&= -2\,E_{\rm o}\,\vec{u}_y\int_{-a/2}^{a/2} e^{j\,k\,u\,x'}\,dx' \int_{-b/2}^{b/2} e^{j\,k\,v\,y'}\,dy' \\
&= -2\,E_{\rm o}\,a\,b\,\mathrm{sinc}(\pi u a/\lambda)\,\mathrm{sinc}(\pi v b/\lambda)\,\vec{u}_y
\end{aligned}$$

$$\mathrm{sinc}(x) = \frac{\sin x}{x}$$

$$\vec{E} = j\,\frac{e^{-jkr}}{2\,\lambda\,r}\;\vec{u}_r \times \vec{L}$$

$$= j\,\frac{e^{-jkr}}{\lambda\,r}\;E_\mathrm{o}\,a\,b\,\mathrm{sinc}(\pi u a/\lambda)\,\mathrm{sinc}(\pi v b/\lambda)\,\sqrt{1-\sin^2\vartheta\,\sin^2\varphi}\;\;\vec{p}$$

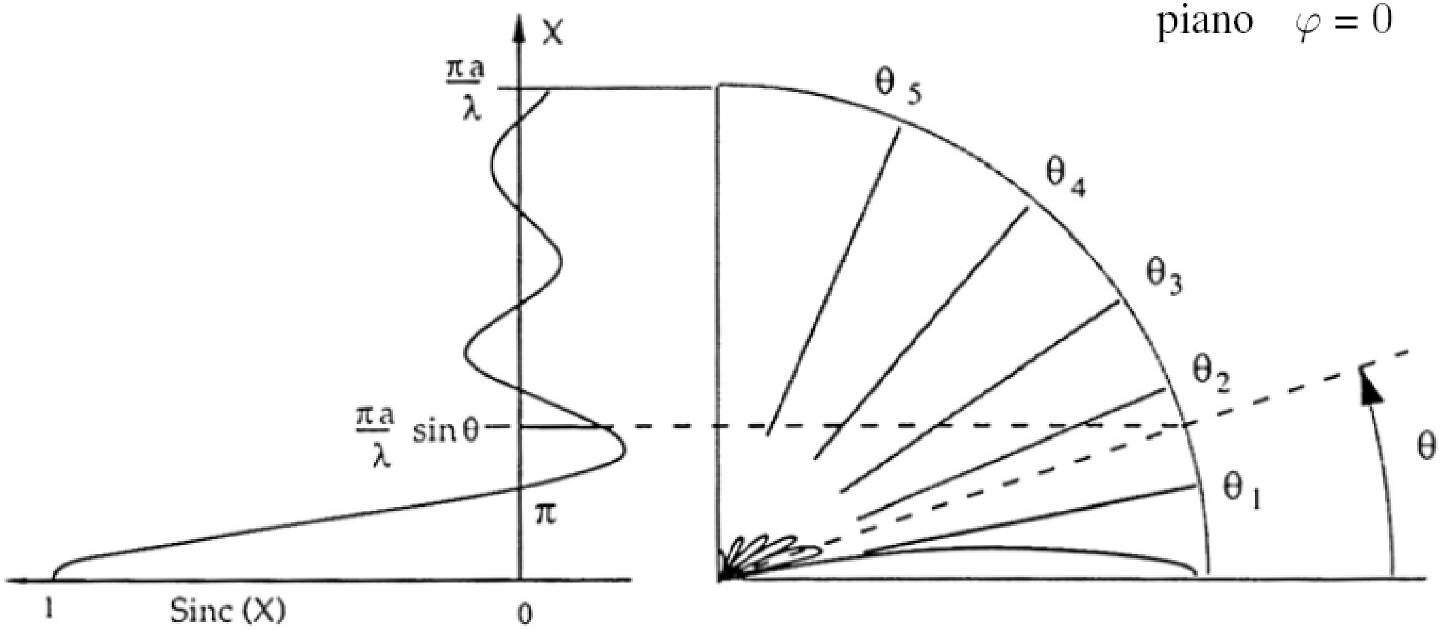
$$\vec{p} = \frac{\vec{u}_\vartheta\cos\varphi - \vec{u}_\varphi\cos\vartheta\,\sin\varphi}{\sqrt{1-\sin^2\vartheta\,\sin^2\varphi}}$$

$$\vec{H} = -\frac{j}{\eta}\,\frac{e^{-jkr}}{2\,\lambda\,r}\;\Big(\vec{L} - \vec{u}_r(\,\vec{u}_r\cdot\vec{L}\,)\Big)$$

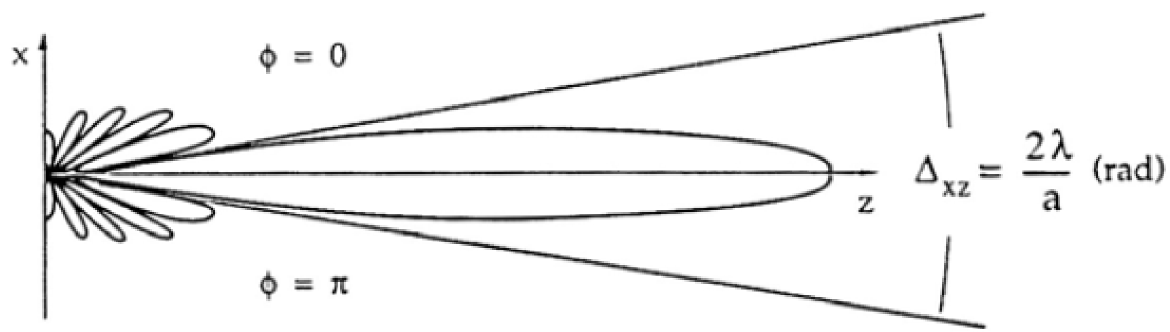
$$= j\,\frac{e^{-jkr}}{\eta\,\lambda\,r}\;E_\mathrm{o}\,a\,b\,\mathrm{sinc}(\pi u a/\lambda)\,\mathrm{sinc}(\pi v b/\lambda)\,\sqrt{1-\sin^2\vartheta\,\sin^2\varphi}\;\;\vec{u}_r\times\vec{p}$$

$$K = \frac{1}{8\,\eta\,\lambda^2}\;\Big(|\,\vec{L}\,|^2 - |\vec{u}_r\cdot\vec{L}\,|^2\Big)$$

$$= \frac{a^2\,b^2}{2\,\eta\,\lambda^2}\;|\,E_\mathrm{o}|^2\;\;\mathrm{sinc}^2(\pi u a/\lambda)\,\mathrm{sinc}^2(\pi v b/\lambda)\,(1-\sin^2\vartheta\,\sin^2\varphi)$$



piano $\varphi = 0$
X
$\frac{\pi a}{\lambda}$
$\frac{\pi a}{\lambda}$ sin θ
π
1
Sinc (X)
0
θ 5
θ 4
θ3
θ2
θ 1
θ



x
ϕ = 0
ϕ = π
z
$\Delta_{xz} = \frac{2\lambda}{a}$ (rad)

Apertura rettangolare illuminata uniformemente con variazione di fase lineare

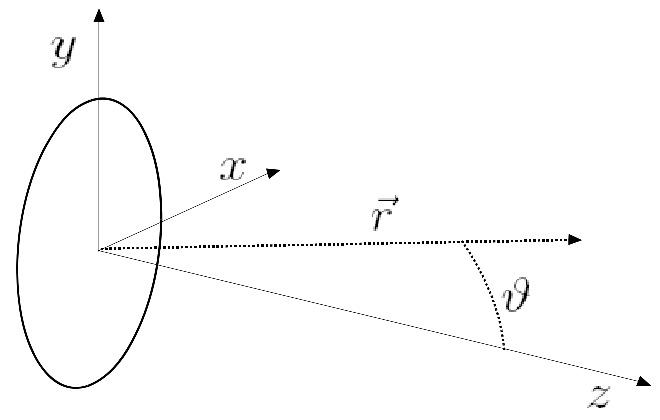
$$\vec{E}_\mathrm{o} = E_\mathrm{o}\vec{u}_x e^{-j\,k(\delta_x x+\delta_y y)}$$

$$\begin{aligned}
\vec{L} &= -2\,E_\mathrm{o}\,\vec{u}_y \int_{-a/2}^{a/2} e^{j\,k(u-\delta_x)\,x'}\,dx' \int_{-b/2}^{b/2} e^{j\,k(v-\delta_y)\,y'}\,dy' \\
&= -2\,E_\mathrm{o}\,a\,b\,\mathrm{sinc}(\pi(u-\delta_x)a/\lambda)\,\mathrm{sinc}(\pi(v-\delta_y)b/\lambda)\,\vec{u}_y \\
\vec{E} &= j\,\frac{e^{-jkr}}{\lambda\,r}\,E_\mathrm{o}\,a\,b\,\mathrm{sinc}(\pi(u-\delta_x)a/\lambda)\,\mathrm{sinc}(\pi(v-\delta_y)b/\lambda)\,\sqrt{1-\sin^2\vartheta\sin^2\varphi}\;\vec{p}
\end{aligned}$$

$$\vec{p} = \frac{\vec{u}_\vartheta\cos\varphi - \vec{u}_\varphi\cos\vartheta\sin\varphi}{\sqrt{1-\sin^2\vartheta\sin^2\varphi}}$$

$$\begin{aligned}
\vec{H} &= j\,\frac{e^{-jkr}}{\eta\,\lambda\,r}\,E_\mathrm{o}\,a\,b\,\mathrm{sinc}(\pi(u-\delta_x)a/\lambda)\,\mathrm{sinc}(\pi(v-\delta_y)b/\lambda)\,\sqrt{1-\sin^2\vartheta\sin^2\varphi}\;\vec{u}_r\times\vec{p} \\
K &= \frac{a^2\,b^2}{2\,\eta\,\lambda^2}\,|\,E_\mathrm{o}|^2\;\mathrm{sinc}^2(\pi(u-\delta_x)a/\lambda)\,\mathrm{sinc}^2(\pi(v-\delta_y)b/\lambda)\,(1-\sin^2\vartheta\sin^2\varphi)
\end{aligned}$$

Apertura circolare illuminata uniformemente $\vec{E}_o = E_o \vec{u}_x$



$$\begin{aligned}
\vec{L} &= 2 \int_{\text{apertura}} e^{j\,k\,\vec{u}_r \cdot \vec{r}\,'} M_s(\vec{r}\,')\, ds' \\
&= -2\, E_o\, \vec{u}_y \int_0^{2\pi} \int_0^{a} e^{j\,k\,\rho \sin\vartheta \cos(\varphi - \varphi')}\, \rho\, d\rho\, d\varphi' \\
&= -4\,\pi\, a^2\, E_o\, \frac{J_1(ka \sin\vartheta)}{ka \sin\vartheta}\, \vec{u}_y
\end{aligned}$$

$$\vec{E} = j\,\frac{e^{-jkr}}{2\,\lambda\,r}\,\vec{u}_r \times \vec{L}$$

$$= j\,\frac{e^{-jkr}}{\lambda\,r}\,E_o\,2\,\pi\,a^2\,\frac{J_1(ka\sin\vartheta)}{ka\sin\vartheta}\,\sqrt{1-\sin^2\vartheta\sin^2\varphi}\;\vec{p}$$

$$\vec{p} = \frac{\vec{u}_\vartheta\cos\varphi - \vec{u}_\varphi\cos\vartheta\sin\varphi}{\sqrt{1-\sin^2\vartheta\sin^2\varphi}}$$

$$\vec{H} = -\frac{j}{\eta}\,\frac{e^{-jkr}}{2\,\lambda\,r}\,\left(\vec{L} - \vec{u}_r(\,\vec{u}_r\cdot\vec{L}\,)\right)$$

$$= j\,\frac{e^{-jkr}}{\eta\,\lambda\,r}\,E_o\,2\,\pi\,a^2\,\frac{J_1(ka\sin\vartheta)}{ka\sin\vartheta}\,\sqrt{1-\sin^2\vartheta\sin^2\varphi}\,\vec{u}_r\times\vec{p}$$

$$K = \frac{1}{8\,\eta\,\lambda^2}\,\left(|\,\vec{L}\,|^2 - |\vec{u}_r\cdot\vec{L}\,|^2\right)$$

$$= \frac{2\,\pi^2\,a^4}{\eta\,\lambda^2}\,|\,E_o|^2\,\left(\frac{J_1(ka\sin\vartheta)}{ka\sin\vartheta}\right)^2\,(1-\sin^2\vartheta\sin^2\varphi)$$