Identità algebriche di uso comune

$$a(\boldsymbol{A}+\boldsymbol{B}) = a\boldsymbol{A}+a\boldsymbol{B} \tag{B.1}$$

$$a(\boldsymbol{A}\cdot\boldsymbol{B}) = (a\boldsymbol{A})\cdot\boldsymbol{B} = \boldsymbol{A}\cdot(a\boldsymbol{B}) \tag{B.2}$$

$$a(\boldsymbol{A}\times\boldsymbol{B}) = (a\boldsymbol{A})\times\boldsymbol{B} = \boldsymbol{A}\times(a\boldsymbol{B}) \tag{B.3}$$

$$\boldsymbol{A}+\boldsymbol{B} = \boldsymbol{B}+\boldsymbol{A} \tag{B.4}$$

$$\boldsymbol{A}\cdot\boldsymbol{B} = \boldsymbol{B}\cdot\boldsymbol{A} \tag{B.5}$$

$$\boldsymbol{A}\times\boldsymbol{B} = -\boldsymbol{B}\times\boldsymbol{A} \tag{B.6}$$

$$\boldsymbol{A}\cdot\boldsymbol{A} = A^2 \tag{B.7}$$

$$\hat{\boldsymbol{u}}\cdot\hat{\boldsymbol{u}} = 1 \tag{B.8}$$

$$\boldsymbol{A}\cdot\hat{\boldsymbol{u}} = A_u \tag{B.9}$$

$$\boldsymbol{A}\cdot(\boldsymbol{B}+\boldsymbol{C}) = \boldsymbol{A}\cdot\boldsymbol{B}+\boldsymbol{A}\cdot\boldsymbol{C} \tag{B.10}$$

$$\boldsymbol{A}\times(\boldsymbol{B}+\boldsymbol{C}) = \boldsymbol{A}\times\boldsymbol{B}+\boldsymbol{A}\times\boldsymbol{C} \tag{B.11}$$

$$\boldsymbol{A}\cdot\boldsymbol{B}\times\boldsymbol{C} = \boldsymbol{C}\cdot\boldsymbol{A}\times\boldsymbol{B} = \boldsymbol{B}\cdot\boldsymbol{C}\times\boldsymbol{A} \tag{B.12}$$

$$\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C}) = \boldsymbol{B}(\boldsymbol{A}\cdot\boldsymbol{C})-\boldsymbol{C}(\boldsymbol{A}\cdot\boldsymbol{B}) \tag{B.13}$$

$$\hat{\boldsymbol{u}}\times(\boldsymbol{A}\times\hat{\boldsymbol{u}}) = \boldsymbol{A}-A_u\hat{\boldsymbol{u}} \tag{B.14}$$

Condizioni di parallelismo, perpendicolarità, complanarità fra vettori

Se $\boldsymbol{A}$, $\boldsymbol{B}$ e $\boldsymbol{C}$ sono vettori non nulli si ha:

$$\boldsymbol{A} \times \boldsymbol{B} = 0 \quad \boldsymbol{A} \text{ e } \boldsymbol{B} \text{ sono paralleli} \tag{B.15}$$

$$\boldsymbol{A} \cdot \boldsymbol{B} = 0 \quad \boldsymbol{A} \text{ e } \boldsymbol{B} \text{ sono perpendicolari} \tag{B.16}$$

$$\boldsymbol{A} \cdot \boldsymbol{B} \times \boldsymbol{C} = 0 \quad \boldsymbol{A}, \boldsymbol{B} \text{ e } \boldsymbol{C} \text{ sono complanari} \tag{B.17}$$

$$\hat{\boldsymbol{n}} \times \boldsymbol{A} = 0 \quad \boldsymbol{A} \text{ è perpendicolare alla superficie orientata } S \text{ di normale } \hat{\boldsymbol{n}} \tag{B.18}$$

$$\hat{\boldsymbol{n}} \cdot \boldsymbol{A} = 0 \quad \boldsymbol{A} \text{ è tangente alla superficie orientata } S \text{ di normale } \hat{\boldsymbol{n}} \tag{B.19}$$

Identità differenziali di uso comune

$$\hat{\boldsymbol{u}} \cdot \nabla F = \frac{\partial F}{\partial u} \tag{B.20}$$

$$\nabla(F + G) = \nabla F + \nabla G \tag{B.21}$$

$$\nabla(FG) = F\nabla G + G\nabla F \tag{B.22}$$

$$\nabla \cdot (\boldsymbol{A} + \boldsymbol{B}) = \nabla \cdot \boldsymbol{A} + \nabla \cdot \boldsymbol{B} \tag{B.23}$$

$$\nabla \cdot (F\boldsymbol{A}) = F\nabla \cdot \boldsymbol{A} + \boldsymbol{A} \cdot \nabla F \tag{B.24}$$

$$\nabla \cdot (\boldsymbol{A} \times \boldsymbol{B}) = \boldsymbol{B} \cdot \nabla \times \boldsymbol{A} - \boldsymbol{A} \cdot \nabla \times \boldsymbol{B} \tag{B.25}$$

$$\nabla \times (\boldsymbol{A} + \boldsymbol{B}) = \nabla \times \boldsymbol{A} + \nabla \times \boldsymbol{B} \tag{B.26}$$

$$\nabla \times (F\boldsymbol{A}) = F\nabla \times \boldsymbol{A} - \boldsymbol{A} \times \nabla F \tag{B.27}$$

$$\nabla \times \nabla F = 0 \tag{B.28}$$

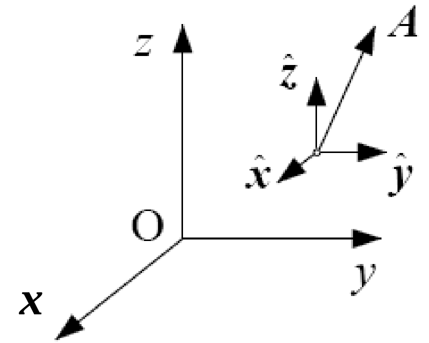
$$\nabla \cdot \nabla \times \boldsymbol{A} = 0 \tag{B.29}$$

$$\nabla^2 F = \nabla \cdot \nabla F \tag{B.30}$$

$$\nabla \times \nabla \times \boldsymbol{A} = \nabla\nabla \cdot \boldsymbol{A} - \nabla^2 \boldsymbol{A} \tag{B.31}$$

$$\nabla^2(F\boldsymbol{A}) = \boldsymbol{A}\nabla^2 F \quad \text{(se } \boldsymbol{A} \text{ è costante)} \tag{B.32}$$

Coordinate cartesiane



$$\boldsymbol{A} = \hat{\boldsymbol{x}}A_x + \hat{\boldsymbol{y}}A_y + \hat{\boldsymbol{z}}A_z$$

$$\boldsymbol{r} = \hat{\boldsymbol{x}}x + \hat{\boldsymbol{y}}y + \hat{\boldsymbol{z}}z$$

$$\hat{\boldsymbol{x}} \cdot \hat{\boldsymbol{x}} = \hat{\boldsymbol{y}} \cdot \hat{\boldsymbol{y}} = \hat{\boldsymbol{z}} \cdot \hat{\boldsymbol{z}} = 1$$

$$\hat{\boldsymbol{x}} \cdot \hat{\boldsymbol{y}} = \hat{\boldsymbol{y}} \cdot \hat{\boldsymbol{z}} = \hat{\boldsymbol{z}} \cdot \hat{\boldsymbol{x}} = 0$$

$$\hat{\boldsymbol{x}} \times \hat{\boldsymbol{y}} = \hat{\boldsymbol{z}} \qquad \hat{\boldsymbol{y}} \times \hat{\boldsymbol{z}} = \hat{\boldsymbol{x}} \qquad \hat{\boldsymbol{z}} \times \hat{\boldsymbol{x}} = \hat{\boldsymbol{y}}$$

$$\boldsymbol{A} + \boldsymbol{B} = \hat{\boldsymbol{x}}(A_x + B_x) + \hat{\boldsymbol{y}}(A_y + B_y) + \hat{\boldsymbol{z}}(A_z + B_z)$$

$$\boldsymbol{A} \cdot \boldsymbol{B} = A_x B_x + A_y B_y + A_z B_z$$

$$A = (\boldsymbol{A} \cdot \boldsymbol{A})^{1/2} = (A_x^2 + A_y^2 + A_z^2)^{1/2}$$

$$\boldsymbol{A} \times \boldsymbol{B} = \hat{\boldsymbol{x}}\,(A_y B_z - A_z B_y) + \hat{\boldsymbol{y}}\,(A_z B_x - A_x B_z) + \hat{\boldsymbol{z}}\,(A_x B_y - A_y B_x)$$

$$\nabla F = \hat{\boldsymbol{x}}\,\frac{\partial F}{\partial x} + \hat{\boldsymbol{y}}\,\frac{\partial F}{\partial y} + \hat{\boldsymbol{z}}\,\frac{\partial F}{\partial z}$$

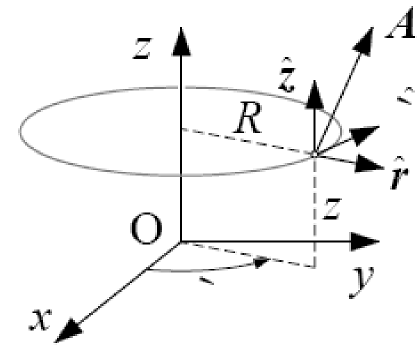
$$\nabla \cdot \boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \boldsymbol{A} = \hat{\boldsymbol{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\boldsymbol{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\boldsymbol{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 F = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2}$$

$$\nabla^2 \boldsymbol{A} = \hat{\boldsymbol{x}}\nabla^2 A_x + \hat{\boldsymbol{y}}\nabla^2 A_y + \hat{\boldsymbol{z}}\nabla^2 A_z$$

Coordinate cilindriche



$$\boldsymbol{A} = \hat{\boldsymbol{R}} A_R + \hat{\boldsymbol{\Phi}} A_\Phi + \hat{\boldsymbol{z}} A_z$$

$$\boldsymbol{r} = \hat{\boldsymbol{R}} R + \hat{\boldsymbol{z}} z$$

$$\hat{\boldsymbol{R}} \cdot \hat{\boldsymbol{R}} = \hat{\boldsymbol{\Phi}} \cdot \hat{\boldsymbol{\Phi}} = \hat{\boldsymbol{z}} \cdot \hat{\boldsymbol{z}} = 1$$

$$\hat{\boldsymbol{R}} \cdot \hat{\boldsymbol{\Phi}} = \hat{\boldsymbol{\Phi}} \cdot \hat{\boldsymbol{z}} = \hat{\boldsymbol{z}} \cdot \hat{\boldsymbol{R}} = 0$$

$$\hat{\boldsymbol{R}} \times \hat{\boldsymbol{\Phi}} = \hat{\boldsymbol{z}} \qquad \hat{\boldsymbol{\Phi}} \times \hat{\boldsymbol{z}} = \hat{\boldsymbol{R}} \qquad \hat{\boldsymbol{z}} \times \hat{\boldsymbol{R}} = \hat{\boldsymbol{\Phi}}$$

$$\boldsymbol{A} + \boldsymbol{B} = \hat{\boldsymbol{R}}(A_R + B_R) + \hat{\boldsymbol{\Phi}}(A_\Phi + B_\Phi) + \hat{\boldsymbol{z}}(A_z + B_z)$$

$$\boldsymbol{A} \cdot \boldsymbol{B} = A_R B_R + A_\Phi B_\Phi + A_z B_z$$

$$A = (\boldsymbol{A} \cdot \boldsymbol{A})^{1/2} = (A_R^2 + A_\Phi^2 + A_z^2)^{1/2}$$

$$\boldsymbol{A} \times \boldsymbol{B} = \hat{\boldsymbol{R}}\left(A_\Phi B_z - A_z B_\Phi\right) + \hat{\boldsymbol{\Phi}}\left(A_z B_R - A_R B_z\right) + \hat{\boldsymbol{z}}\left(A_R B_\Phi - A_\Phi B_R\right)$$

$$\nabla F = \hat{\boldsymbol{R}} \frac{\partial F}{\partial R} + \hat{\boldsymbol{\Phi}} \frac{1}{R} \frac{\partial F}{\partial \phi} + \hat{\boldsymbol{z}} \frac{\partial F}{\partial z}$$

$$\nabla \cdot \boldsymbol{A} = \frac{1}{R} \frac{\partial (R A_R)}{\partial R} + \frac{1}{R} \frac{\partial A_\Phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \boldsymbol{A} = \hat{\boldsymbol{R}} \left(\frac{1}{R} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\Phi}{\partial z} \right) + \hat{\boldsymbol{\Phi}} \left(\frac{\partial A_R}{\partial z} - \frac{\partial A_z}{\partial R} \right) + \hat{\boldsymbol{z}} \frac{1}{R} \left(\frac{\partial (R A_\Phi)}{\partial R} - \frac{\partial A_R}{\partial \phi} \right)$$

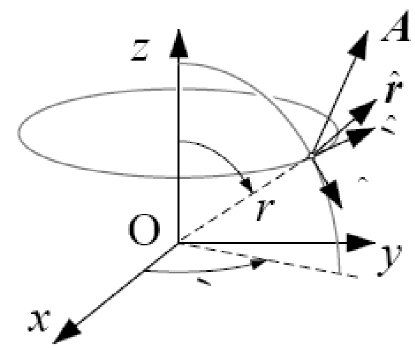
$$\nabla^2 F = \frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial F}{\partial R} \right) + \frac{1}{R^2} \frac{\partial^2 F}{\partial \phi^2} + \frac{\partial^2 F}{\partial z^2}$$

$$\nabla^2 \boldsymbol{A} = \hat{\boldsymbol{R}} \left(\nabla^2 A_R - \frac{A_R}{R^2} - \frac{2}{R^2} \frac{\partial A_\Phi}{\partial \phi} \right) + \hat{\boldsymbol{\Phi}} \left(\nabla^2 A_\Phi - \frac{A_\Phi}{R^2} - \frac{2}{R^2} \frac{\partial A_R}{\partial \phi} \right) + \hat{\boldsymbol{z}} \nabla^2 A_z$$

Coordinate sferiche

$$\boldsymbol{A} = \hat{\boldsymbol{r}} A_r + \hat{\boldsymbol{\theta}} A_\theta + \hat{\boldsymbol{\phi}} A_\phi$$

$$\boldsymbol{r} = \hat{\boldsymbol{r}} r$$



$$\boldsymbol{A} \cdot \boldsymbol{B} = A_r B_r + A_\theta B_\theta + A_\phi B_\phi$$

$$A = (\boldsymbol{A} \cdot \boldsymbol{A})^{1/2} = (A_r^2 + A_\theta^2 + A_\phi^2)^{1/2}$$

$$\boldsymbol{A} \times \boldsymbol{B} = \hat{\boldsymbol{r}} \left(A_\theta B_\phi - A_\phi B_\theta \right) + \hat{\boldsymbol{\theta}} \left(A_\phi B_r - A_r B_\phi \right) + \hat{\boldsymbol{\phi}} \left(A_r B_\theta - A_\theta B_r \right)$$

$$\nabla F = \hat{\boldsymbol{r}} \frac{\partial F}{\partial r} + \hat{\boldsymbol{\theta}} \frac{1}{r} \frac{\partial F}{\partial \theta} + \hat{\boldsymbol{\phi}} \frac{1}{r \sin\theta} \frac{\partial F}{\partial \phi}$$

$$\nabla \cdot \boldsymbol{A} = \frac{1}{r^2} \frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial (\sin\theta A_\theta)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \boldsymbol{A} = \hat{\boldsymbol{r}} \frac{1}{r \sin\theta} \left(\frac{\partial (\sin\theta A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right) + \hat{\boldsymbol{\theta}} \frac{1}{r} \left(\frac{1}{\sin\theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial (r A_\phi)}{\partial r} \right) + \hat{\boldsymbol{\phi}} \frac{1}{r} \left(\frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right)$$

$$\nabla^2 F = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial F}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial F}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 F}{\partial \phi^2}$$

$$\nabla^2 \boldsymbol{A} = \hat{\boldsymbol{r}} \left[\nabla^2 A_r - \frac{2}{r^2} \left(A_r + \frac{1}{\tan\theta} A_\theta + \frac{\partial A_\theta}{\partial \theta} + \frac{1}{\sin\theta} \frac{\partial A_\phi}{\partial \phi} \right) \right]$$

$$+ \hat{\boldsymbol{\theta}} \left[\nabla^2 A_\theta - \frac{1}{r^2} \left(\frac{1}{\sin^2\theta} A_\theta - 2 \frac{\partial A_r}{\partial \theta} + 2 \frac{\cos\theta}{\sin^2\theta} \frac{\partial A_\phi}{\partial \phi} \right) \right]$$

$$+ \hat{\boldsymbol{\phi}} \left[\nabla^2 A_\phi - \frac{1}{r^2} \left(\frac{1}{\sin^2\theta} A_\phi - 2 \frac{1}{\sin\theta} \frac{\partial A_r}{\partial \phi} - 2 \frac{\cos\theta}{\sin^2\theta} \frac{\partial A_\theta}{\partial \phi} \right) \right]$$

Trasformazione fra coordinate cartesiane e coordinate cilindriche

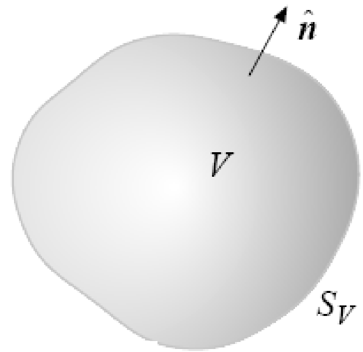
$$\begin{aligned}\hat{\boldsymbol{R}} &= \hat{\boldsymbol{x}}\cos\phi + \hat{\boldsymbol{y}}\sin\phi \\ \hat{\boldsymbol{\Phi}} &= -\hat{\boldsymbol{x}}\sin\phi + \hat{\boldsymbol{y}}\cos\phi\end{aligned}$$

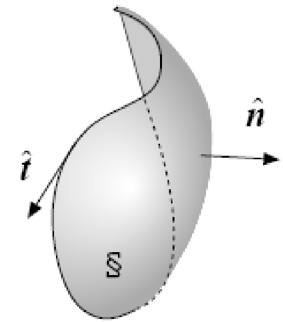
$$\begin{aligned}\hat{\boldsymbol{x}} &= \hat{\boldsymbol{R}}\cos\phi - \hat{\boldsymbol{\Phi}}\sin\phi \\ \hat{\boldsymbol{y}} &= \hat{\boldsymbol{R}}\sin\phi + \hat{\boldsymbol{\Phi}}\cos\phi\end{aligned}$$

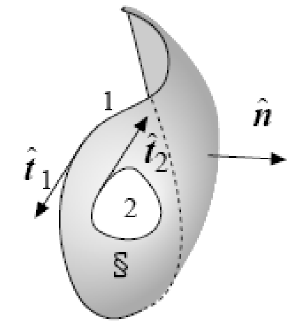
Trasformazione fra coordinate cartesiane e coordinate sferiche

$$\begin{aligned}\hat{\boldsymbol{r}} &= \hat{\boldsymbol{x}}\sin\theta\cos\phi + \hat{\boldsymbol{y}}\sin\theta\sin\phi + \hat{\boldsymbol{z}}\cos\theta \\ \hat{\boldsymbol{\theta}} &= \hat{\boldsymbol{x}}\cos\theta\cos\phi + \hat{\boldsymbol{y}}\cos\theta\sin\phi + \hat{\boldsymbol{z}}\sin\theta \\ \hat{\boldsymbol{\phi}} &= \hat{\boldsymbol{x}}\sin\phi - \hat{\boldsymbol{y}}\cos\phi\end{aligned}$$

$$\begin{aligned}\hat{\boldsymbol{x}} &= \hat{\boldsymbol{r}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi \\ \hat{\boldsymbol{y}} &= \hat{\boldsymbol{r}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi \\ \hat{\boldsymbol{z}} &= \hat{\boldsymbol{r}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta\end{aligned}$$







Teorema della divergenza o di Gauss

$$\int_V \nabla \cdot \boldsymbol{A}\, dV = \int_{S_V} \hat{\boldsymbol{n}} \cdot \boldsymbol{A}\, dS_V$$

Teorema di Stokes

$$\int_\Sigma \hat{\boldsymbol{n}} \cdot \nabla \times \boldsymbol{A}\, dS = \int_\sigma \hat{\boldsymbol{t}} \cdot \boldsymbol{A}\, d\ell$$

Formula del rotore

$$\int_V \nabla \times \boldsymbol{A}\, dV = \int_{S_V} \hat{\boldsymbol{n}} \times \boldsymbol{A}\, dS_V$$

Formula del gradiente

$$\int_V \nabla F\, dV = \int_{S_V} F \hat{\boldsymbol{n}}\, dS_V$$

Integrale di un campo conservativo lungo una linea orientata dall'estremo P all'estremo Q

$$\int_{\mathrm{P}}^{\mathrm{Q}} \nabla F \cdot \hat{\boldsymbol{t}}\, d\ell = F_{\mathrm{Q}} - F_{\mathrm{P}}$$

Circuitazione di un campo conservativo

$$\oint_{\sigma} \nabla F \cdot \hat{\boldsymbol{t}}\, d\ell = 0$$

Prima formula di Green

$$\int_V \left(G\nabla^2 F + \nabla F \cdot \nabla G\right) dV = \int_{S_V} G\, \frac{\partial F}{\partial n}\, dS_V$$

Seconda formula di Green

$$\int_V \left(G\nabla^2 F - F\nabla^2 G\right) dV = \int_{S_V} \left(G\, \frac{\partial F}{\partial n} - F\, \frac{\partial G}{\partial n}\right) dS_V$$