Facoltà di Ingegneria
Università degli studi di Pavia

Corso di Laurea Triennale in
Ingegneria Elettronica e Informatica

Circuiti Elettrici Lineari

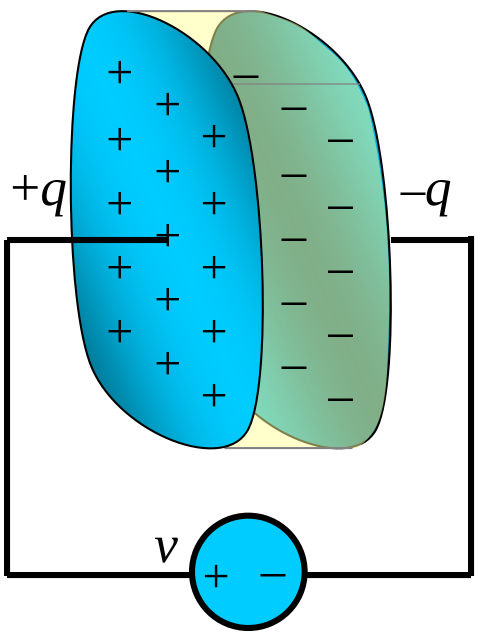
Condensatori e induttori

Sommario

- Il Condensatore
- Tensione, corrente, potenza ed energia nel condensatore
- Condensatori in serie e in parallelo
- L'induttore
- Tensione, corrente, potenza ed energia nell'induttore
- Induttori in serie e in parallelo

Condensatore: relazione tensione/carica

La carica sulle armature è proporzionale alla tensione ai loro capi attraverso il valore di capacità del condensatore

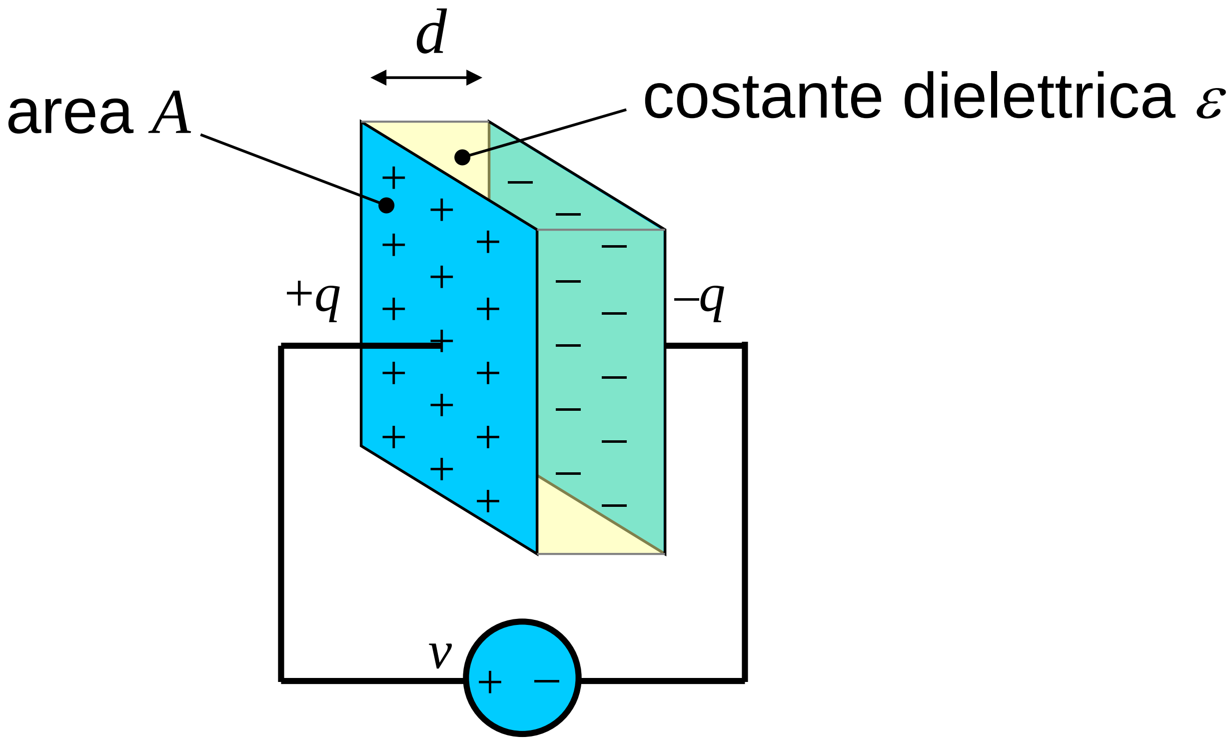


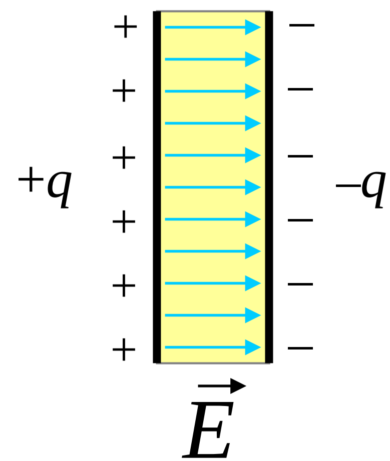
$$q = C \cdot v$$

La capacità si misura in farad (F) in onore di Michael Faraday (1791-1867)

$$1\ \mathrm{F} = 1\ \mathrm{C/V} = 1\ \mathrm{A \cdot s/V}$$

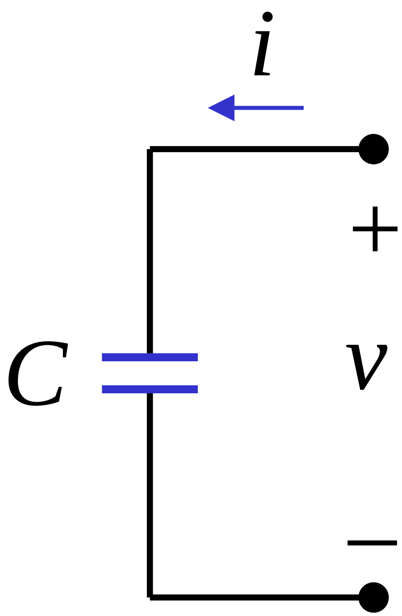
Condensatore a piani paralleli





$$C = \varepsilon \cdot \frac{A}{d}$$

Condensatore: relazione tensione/corrente

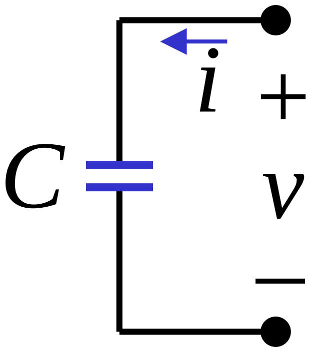


$$i = \frac{dq}{dt} = \frac{d(C \cdot v)}{dt} = C \cdot \frac{dv}{dt}$$

(con la convenzione degli utilizzatori)

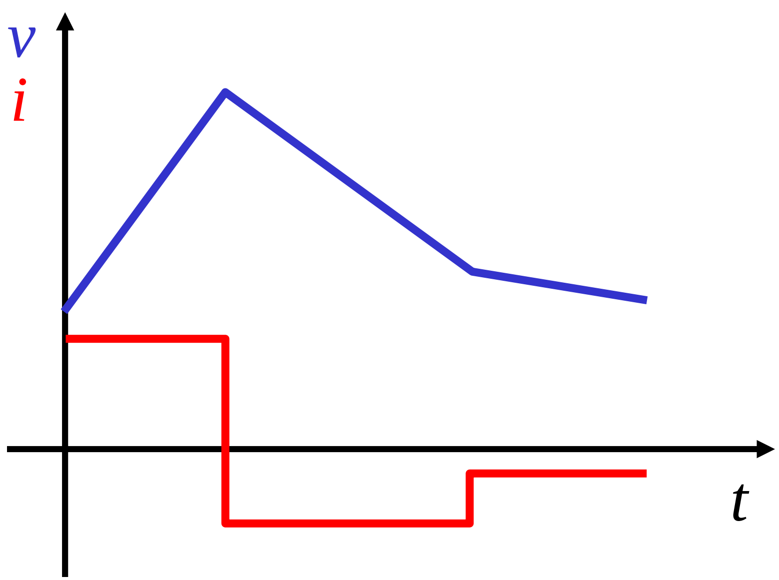
$$v(t) = \frac{1}{C} \int_{t_0}^{t} i(t)\, dt + v(t_0)$$

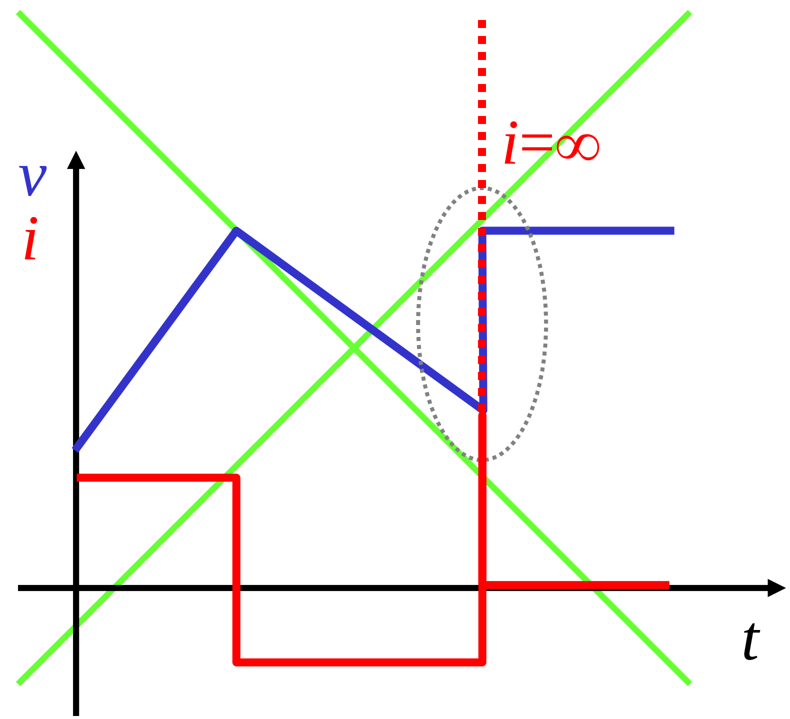
Condensatore: relazione tensione/corrente



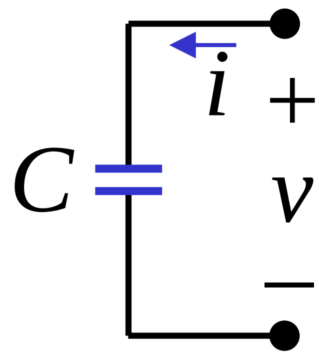
$$i = C \cdot \frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt + v(t_0)$$



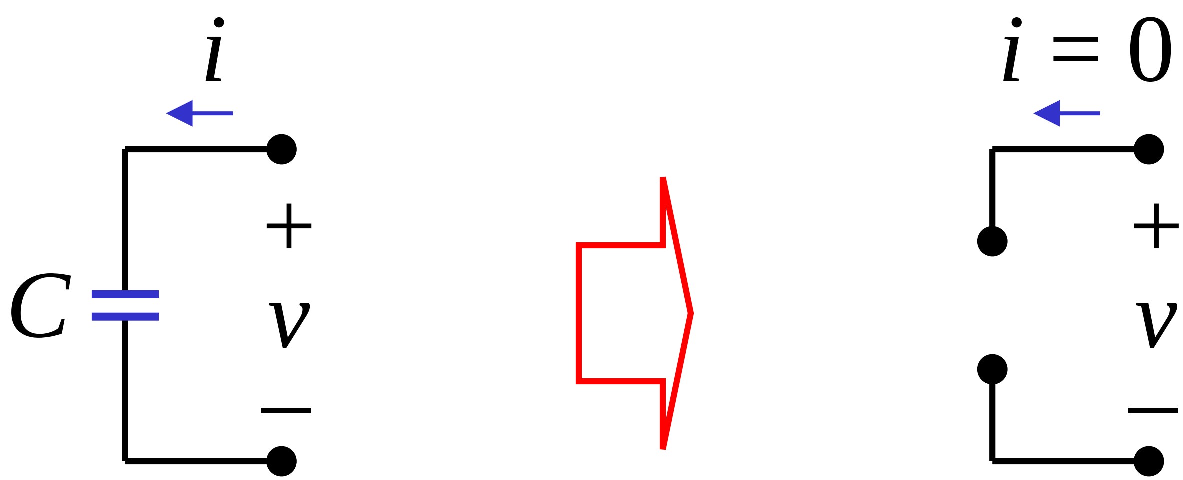


Condensatore in regime stazionario



$$i = C \cdot \frac{dv}{dt}$$

v costante nel tempo $\Rightarrow$ $i = 0$ (circuito aperto)



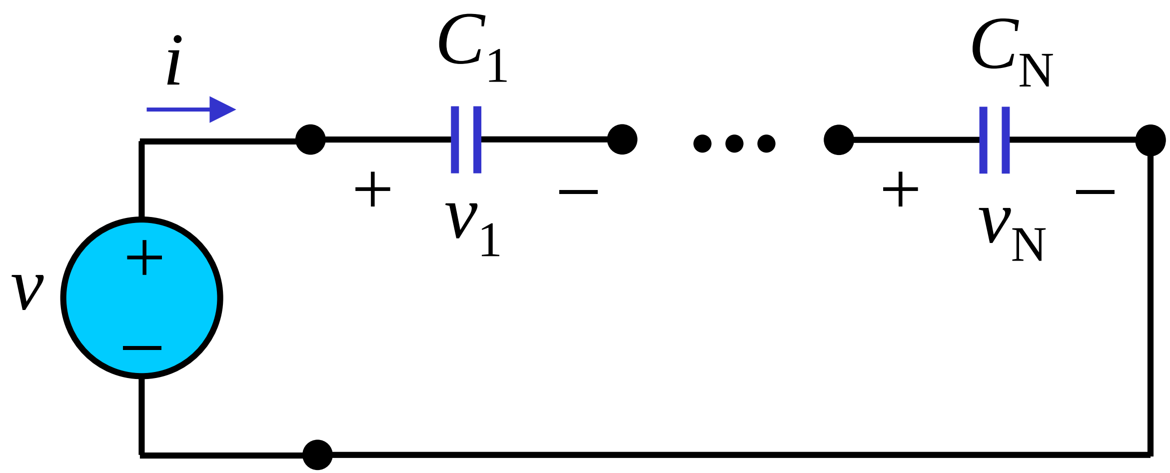
Condensatore: potenza ed energia

$$p = v \cdot i = C \cdot v \cdot \frac{dv}{dt}$$

$$w = \int_{-\infty}^{t} p(t)\, dt = C \int_{-\infty}^{t} v \cdot \frac{dv}{dt}\, dt = C \int_{v(-\infty)}^{v(t)} v\, dv =$$

$$= \frac{1}{2} C \cdot v^2 \Big|_{0}^{v(t)} = \frac{1}{2} C \cdot v^2 = \frac{q^2}{2C}$$

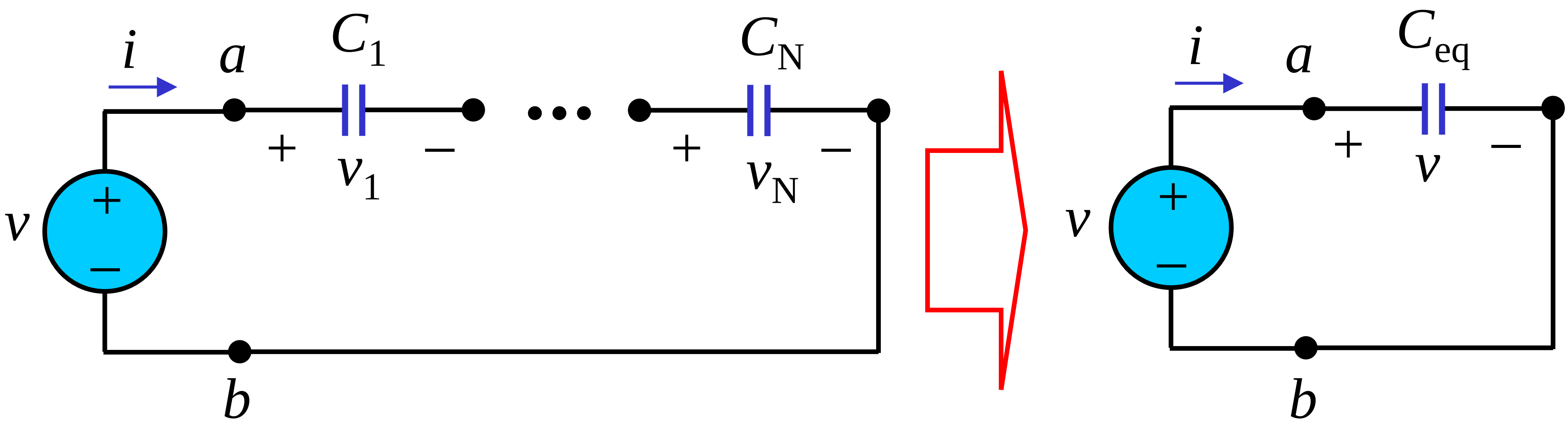
Condensatori in serie



$$v = \frac{1}{C_1}\int_{t_0}^{t} i\,dt + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i\,dt + v_2(t_0) + \ldots + \frac{1}{C_\mathrm{N}}\int_{t_0}^{t} i\,dt + v_\mathrm{N}(t_0) =$$

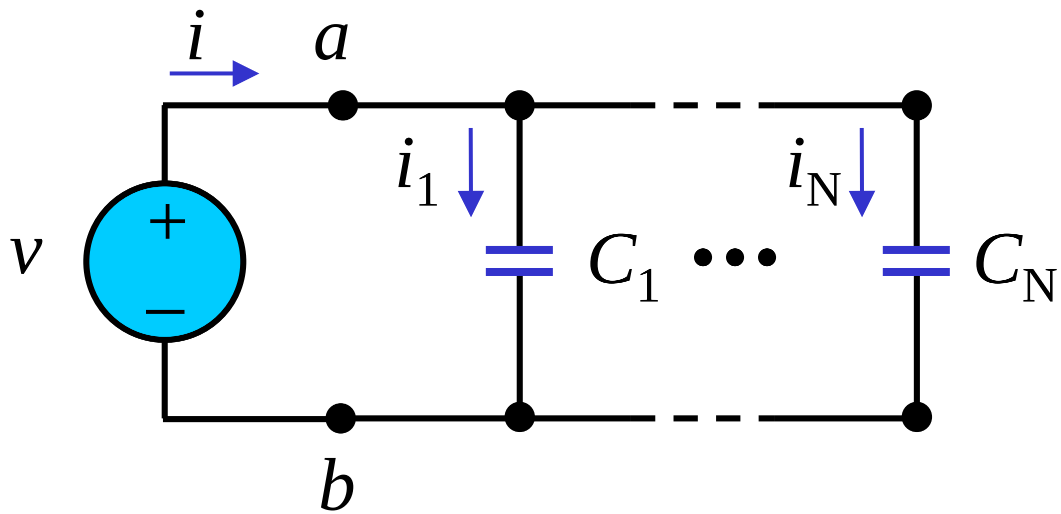
$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_\mathrm{N}}\right)\int_{t_0}^{t} i\,dt + v_1(t_0) + v_2(t_0) + \ldots + v_\mathrm{N}(t_0) = \frac{1}{C_\mathrm{eq}}\int_{t_0}^{t} i\,dt + v(t_0)$$

Condensatori in serie



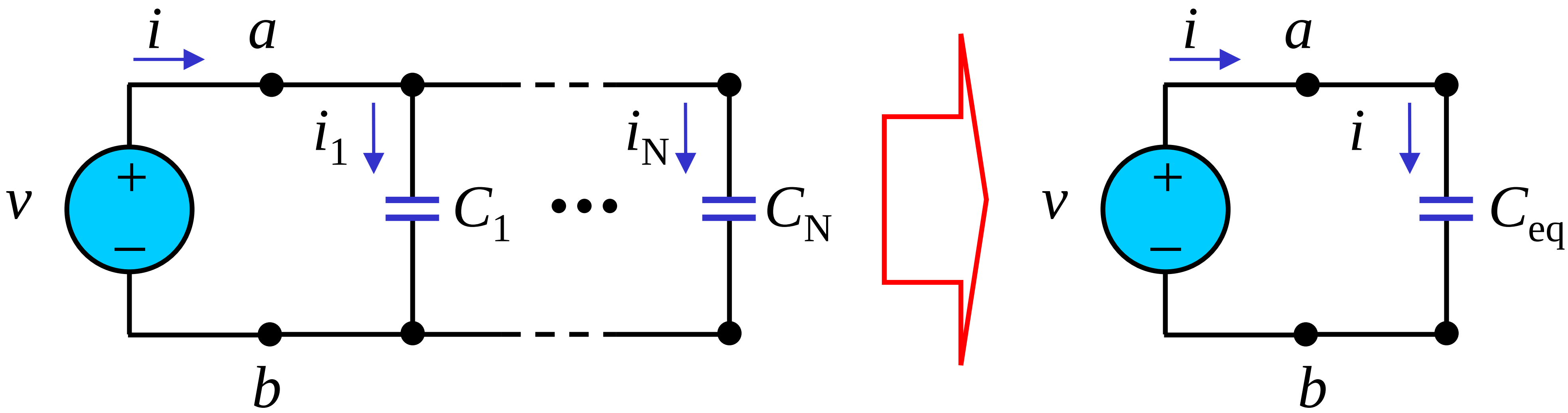
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$$

Condensatori in parallelo



$$i = C_1 \cdot \frac{dv}{dt} + C_2 \cdot \frac{dv}{dt} + \ldots + C_N \cdot \frac{dv}{dt} = \left(C_1 + C_2 + \ldots + C_N\right) \cdot \frac{dv}{dt} = C_{eq} \cdot \frac{dv}{dt}$$

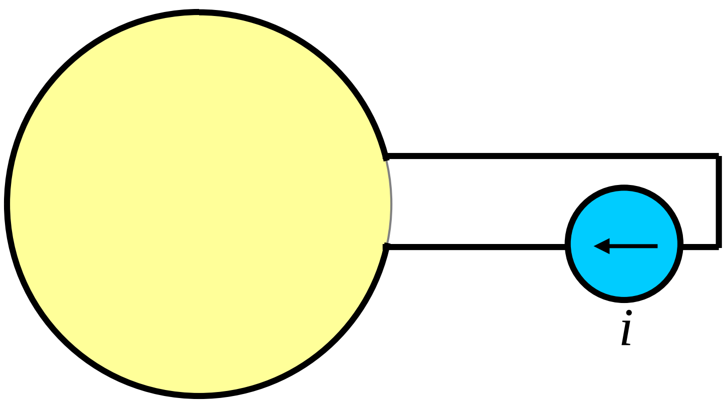
Condensatori in parallelo

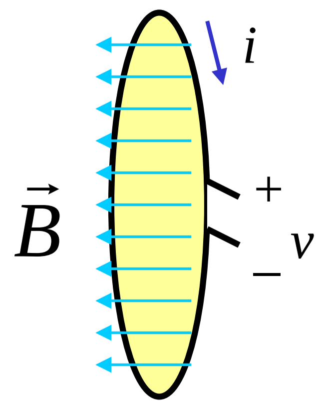


$$C_{eq} = C_1 + C_2 + \ldots + C_N$$

Induttore

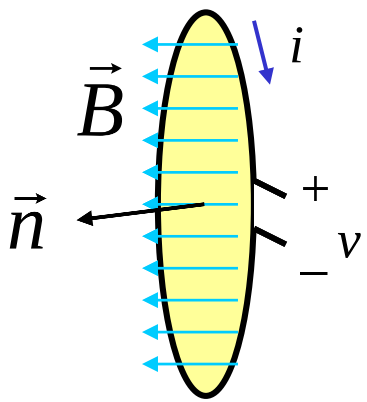
Un **induttore** consiste di un filo (tipicamente avvolto in più spire) percorso da corrente ed è in grado di accumulare energia magnetica





Induttore: relazione flusso/corrente

Il flusso del campo magnetico è proporzionale alla corrente attraverso il valore di induttanza dell'induttore che si misura in Henry (H)



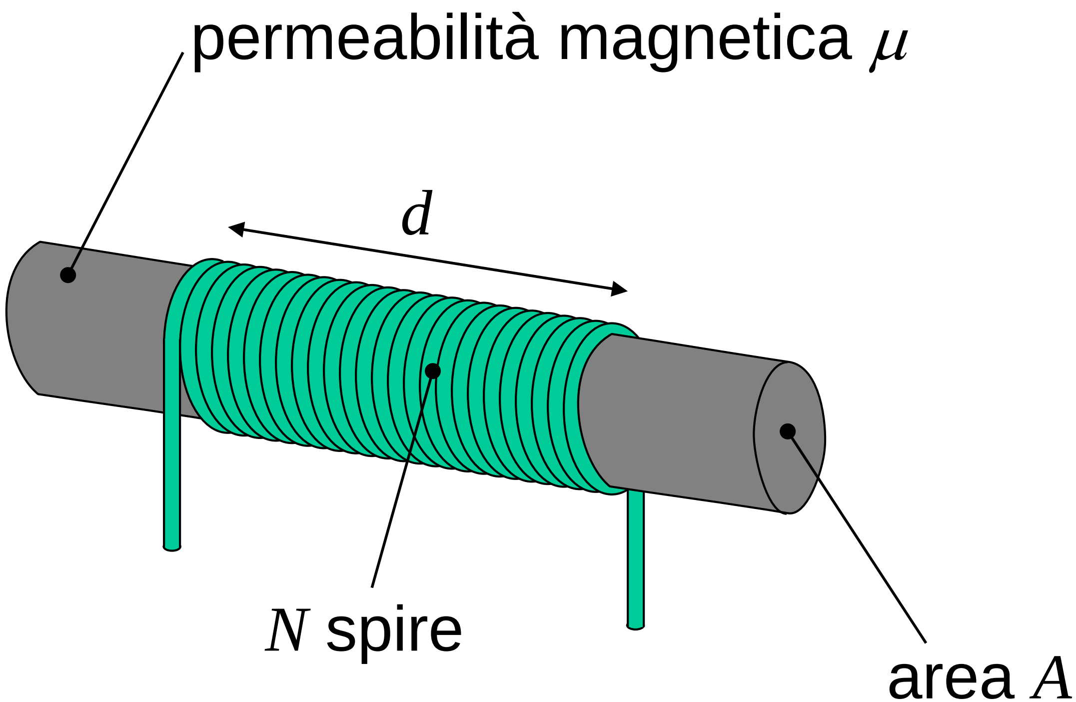
$$\Phi = L \cdot i$$

L'induttanza si misura in henry (H) in onore di Joseph Henry (1797–1878)

$$\Phi = \int_S \vec{B} \cdot \vec{n} \, dS$$

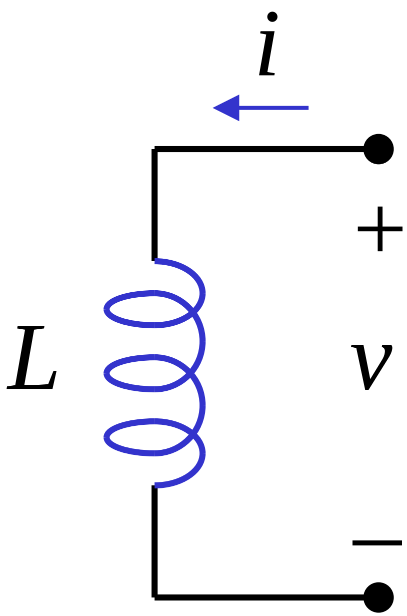
$$1 \text{ H} = 1 \text{ V} \cdot \text{s/A}$$

Induttore a bobina



$$L = \mu \cdot \frac{N^2 A}{d}$$

Induttore: relazione tensione/corrente

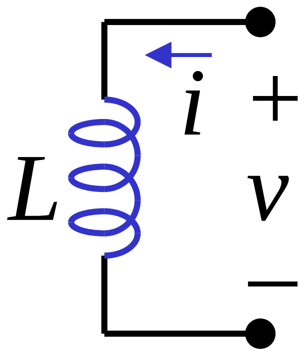


$$v = \frac{d\Phi}{dt} = \frac{d(L \cdot i)}{dt} = L \cdot \frac{di}{dt}$$

(con la convenzione degli utilizzatori)

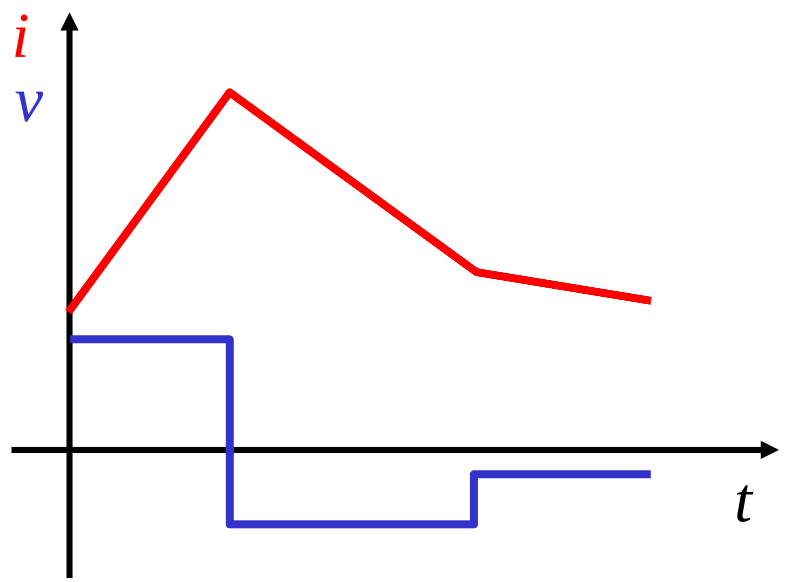
$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\, dt + i(t_0)$$

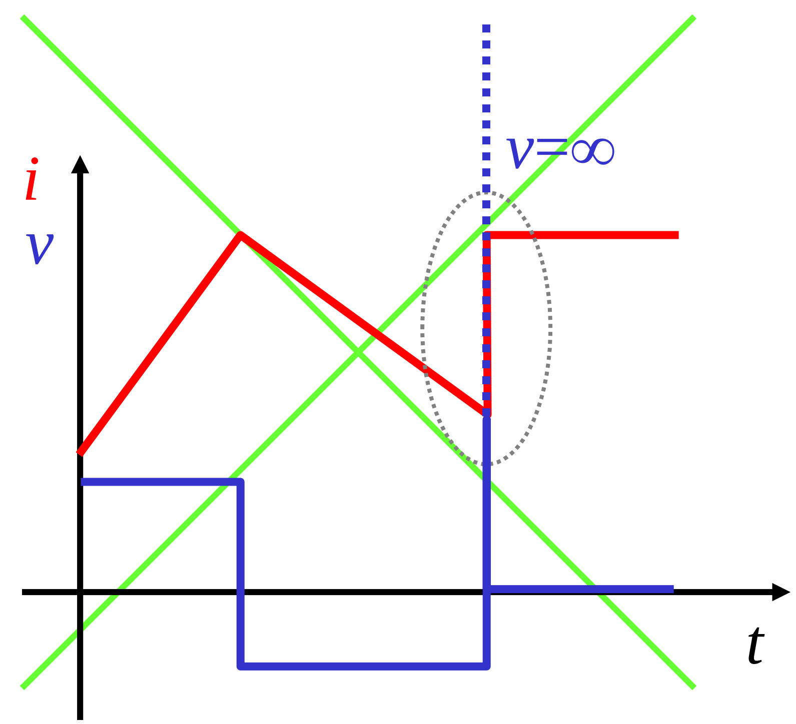
Induttore: relazione tensione/corrente



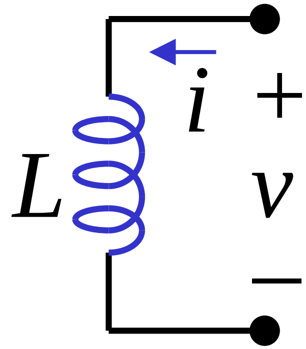
$$v = L \cdot \frac{di}{dt}$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$



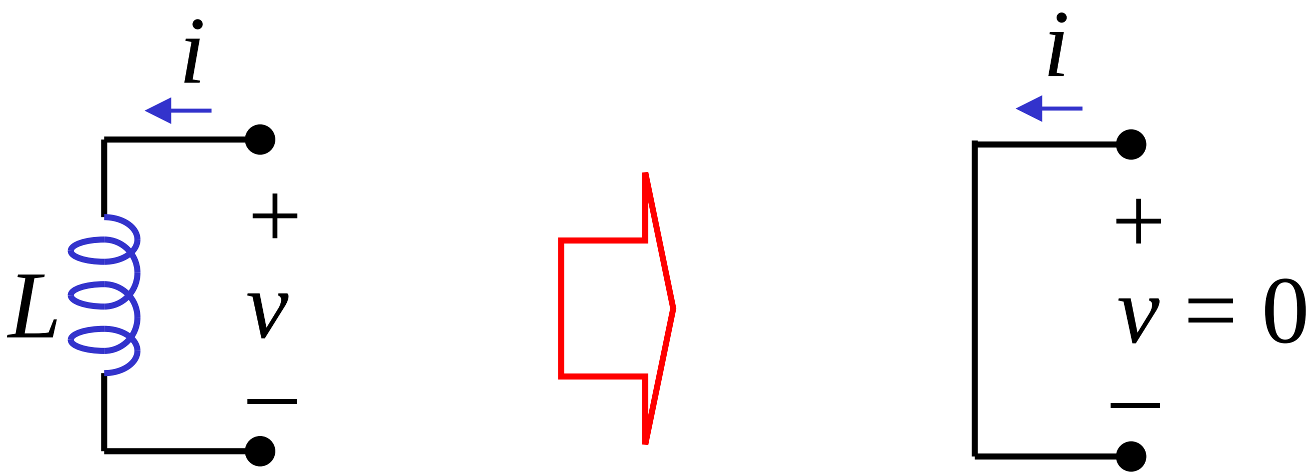


Induttore in regime stazionario



$$v = L \cdot \frac{di}{dt}$$

i costante nel tempo ⇒ $v = 0$ (corto circuito)



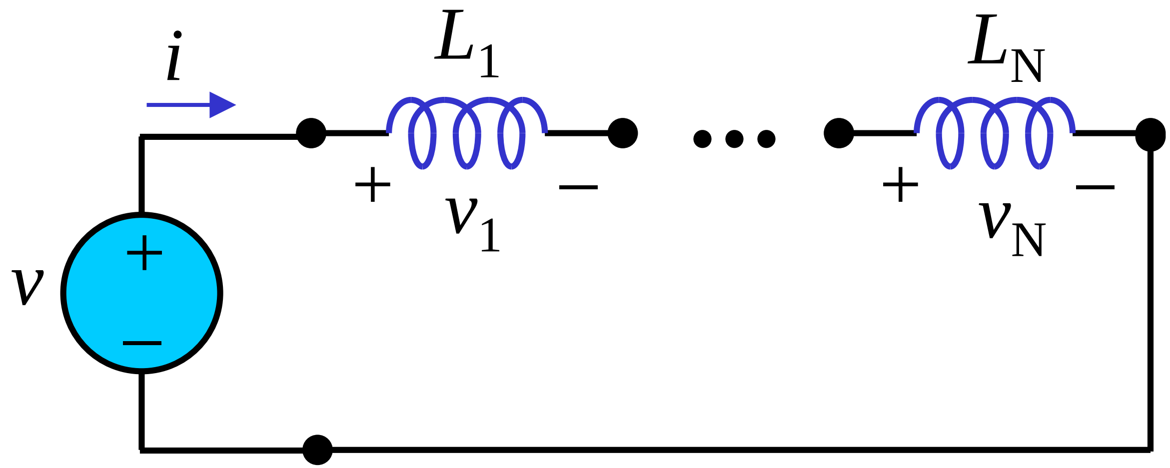
Induttore: potenza ed energia

$$p = v \cdot i = L \cdot \frac{di}{dt} \cdot i$$

$$w = \int_{-\infty}^{t} p(t)\, dt = L \int_{-\infty}^{t} i \cdot \frac{di}{dt}\, dt = L \int_{i(-\infty)}^{i(t)} i\, di =$$

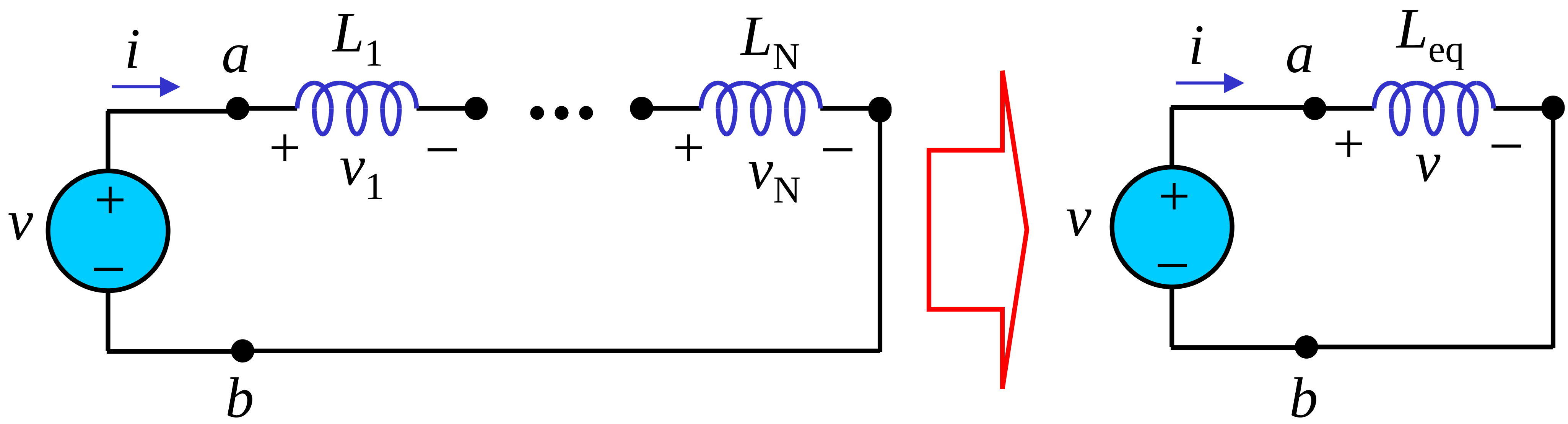
$$= \frac{1}{2} L \cdot i^2 \Big|_0^{i(t)} = \frac{1}{2} L \cdot i^2$$

Induttori in serie



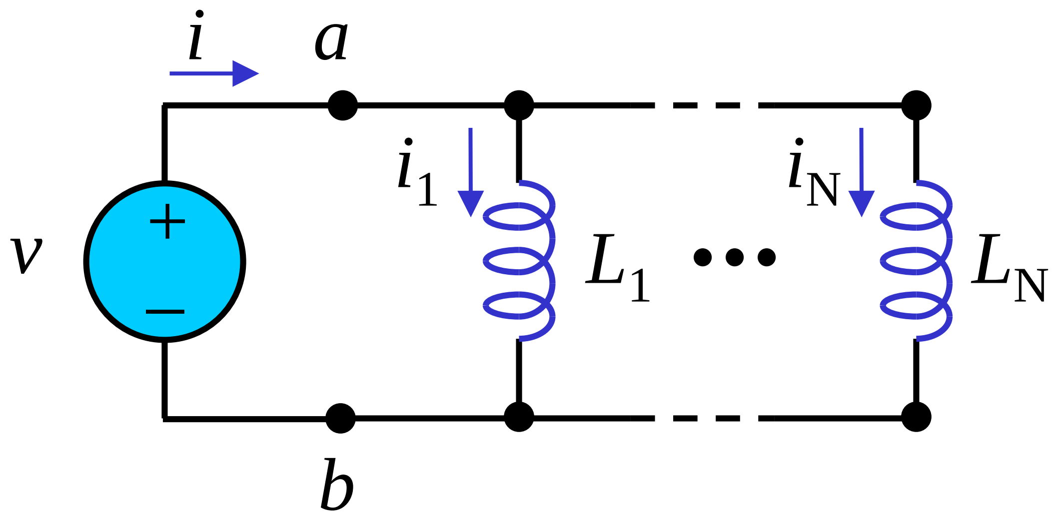
$$v = L_1 \cdot \frac{di}{dt} + L_2 \cdot \frac{di}{dt} + \ldots + L_N \cdot \frac{di}{dt} = \left(L_1 + L_2 + \ldots + L_N\right) \cdot \frac{di}{dt} = L_{eq} \cdot \frac{di}{dt}$$

Induttori in serie



$$L_{eq} = L_1 + L_2 + \ldots + L_N$$

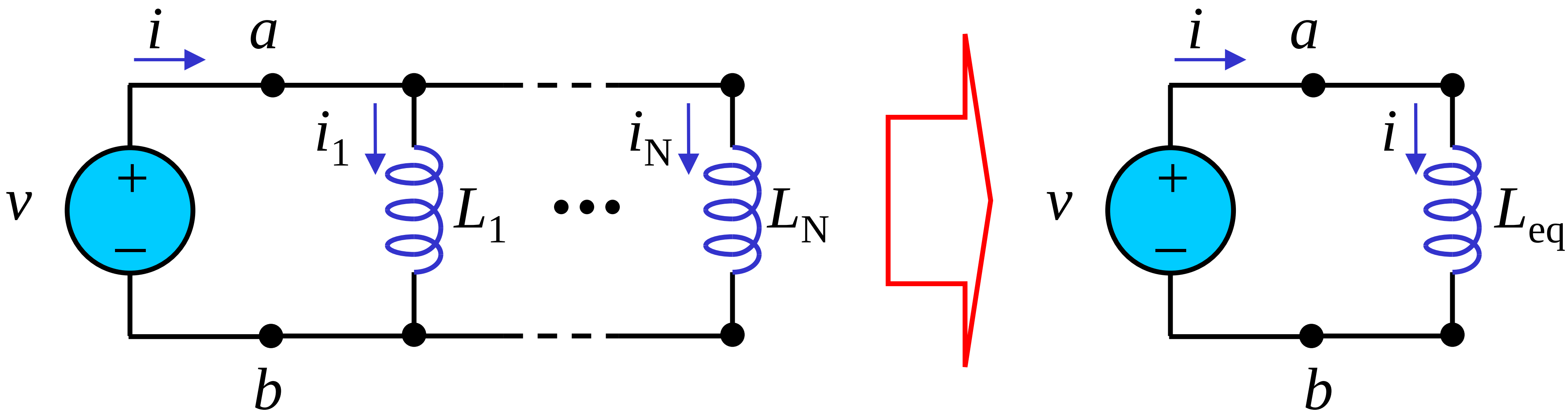
Induttori in parallelo



$$i = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v\,dt + i_2(t_0) + \ldots + \frac{1}{L_N}\int_{t_0}^{t} v\,dt + i_N(t_0) =$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}\right)\int_{t_0}^{t} v\,dt + i_1(t_0) + i_2(t_0) + \ldots + i_N(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt + i(t_0)$$

Induttori in parallelo



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}$$