



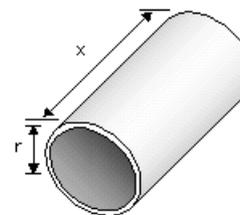
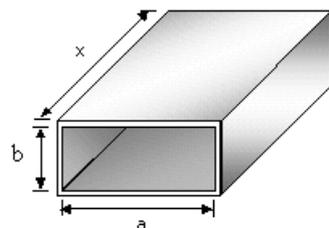
Lecture 13

WIDEBAND MODELING OF WAVEGUIDE COMPONENTS BY THE BI-RME METHOD

- Classical methods for calculating **arbitrarily-shaped waveguide modes**
- Calculation of **arbitrarily-shaped waveguide** (and cavity) **modes** by the Boundary Integral-Resonant Mode Expansion (**BI-RME**) **method**
- BI-RME modeling of **planar** and **3D waveguide circuits**
- **Fast optimization** of waveguide components
- Conclusions

- Classical methods for calculating **arbitrarily-shaped waveguide modes**
- Calculation of arbitrarily-shaped waveguide (and cavity) modes by the Boundary Integral-Resonant Mode Expansion (BI-RME) method
- BI-RME modeling of planar and 3D waveguide circuits
- Fast optimization of waveguide components
- Conclusions

Waveguide modes are known analytically only for waveguides with **canonical cross-section** (like rectangular or circular waveguides).



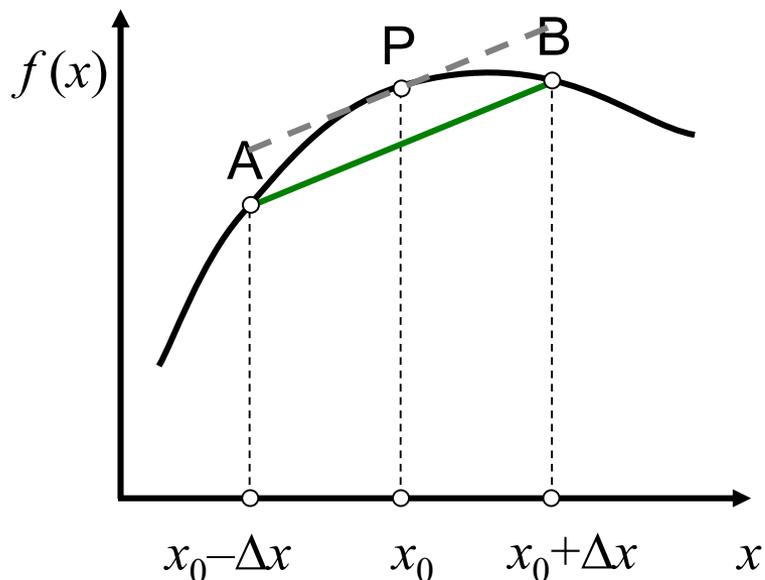
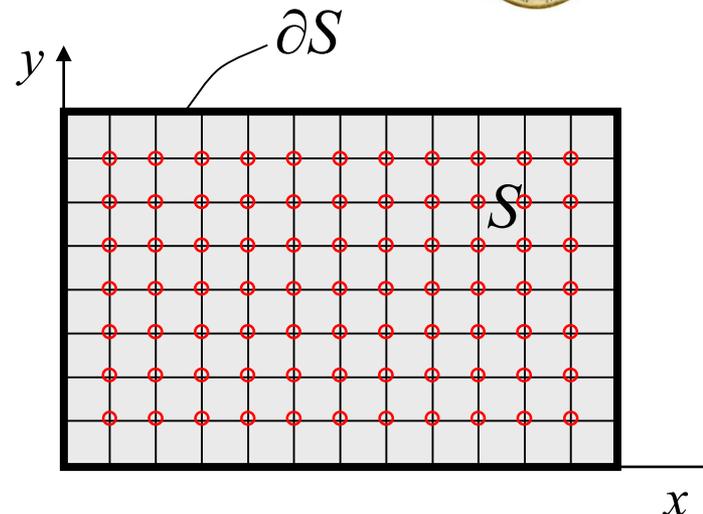
For arbitrarily-shaped waveguides, the mode spectrum is computed numerically, by solving an **eigenvalue problem**.

The eigenvalue problem can be formulated in different ways, starting from the **Helmoltz equations** with proper boundary condition.

$$\left\{ \begin{array}{ll} \nabla_{\text{T}}^2 \Psi - k^2 \Psi = 0 & \text{in } S \\ \Psi = 0 & \text{su } \partial S \end{array} \right. \quad (\text{TM modes})$$

$$\left\{ \begin{array}{ll} \nabla_{\text{T}}^2 \Phi - k^2 \Phi = 0 & \text{in } S \\ d\Phi/dn = 0 & \text{su } \partial S \end{array} \right. \quad (\text{TE modes})$$

The finite difference (FD) method requires the **definition of a grid** in the entire domain of the cross-section.



In each point of the grid, derivatives are approximated by finite differences (**central difference approach**):

$$f'(x_0) \cong \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x)}{2\Delta x}$$

$$f''(x_0) \cong \frac{f'(x_0 + \Delta x/2) - f'(x_0 - \Delta x/2)}{\Delta x}$$

By applying the finite difference method to the Helmholtz equations, with **grid** step $\Delta x = \Delta y = h$ it results:

$$\Phi(i+1, j) + \Phi(i-1, j) + \Phi(i, j+1) + \Phi(i, j-1) - (4 - h^2 k^2) \Phi(i, j) = 0$$

where the nodes are numbered i and j in the x and y directions, respectively.

After applying the **boundary conditions**, the following **eigenvalue problem** is obtained:

$$(\mathbf{A} - \lambda \mathbf{I}) \Phi = 0$$

which provides mode cutoff frequencies (as **eigenvalues**) and modal scalar potentials at the nodes (as **eigenvectors**).



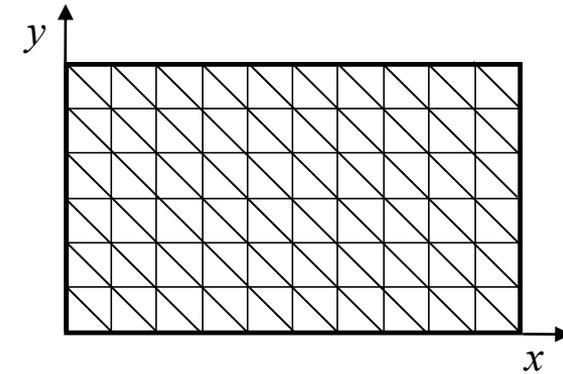
ADVANTAGES

- simple to implement
- a single linear eigenvalue problem provides all modes

DISADVANTAGES

- fine surface grid (**large number of unknowns**)
- cumbersome when higher-order modes are required and the geometry exhibits sharp corners.

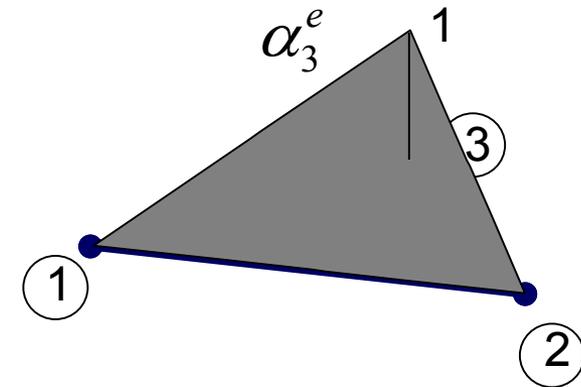
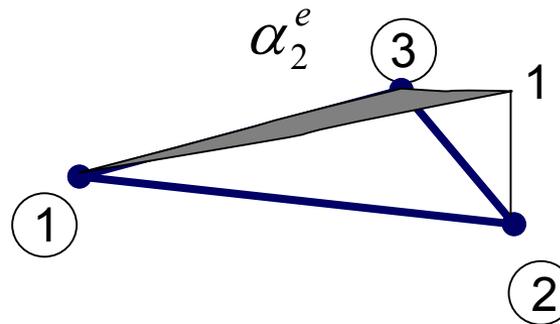
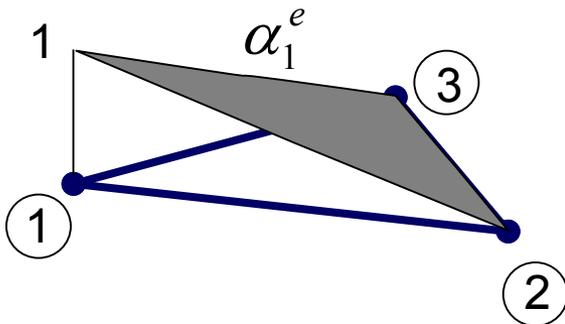
The FEM requires the **segmentation of the entire domain** of the cross-section in small elements, usually triangles.



The unknown function (scalar potential) in each element is expressed as a combination of **interpolating functions**:

$$\sum_{i=1}^3 \alpha_i^e(x, y) = 1$$

$$\alpha_i^e(x_j^e, y_j^e) = \begin{cases} 1 & \text{se } i = j \\ 0 & \text{se } i \neq j \end{cases}$$



The problem is formulated through the **minimization of an integral functional**:

$$F(\phi) = \frac{1}{2} \int_{\Omega} \left[\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 - k^2 \phi \right] d\Omega$$

which finally leads to an **eigenvalue problem** in the form:

$$[A][\phi] = k^2 [B][\phi]$$

This problem is similar to the one obtained using the FD method and provides mode cutoff frequencies (as **eigenvalues**) and modal scalar potentials at the nodes (as **eigenvectors**).



ADVANTAGES

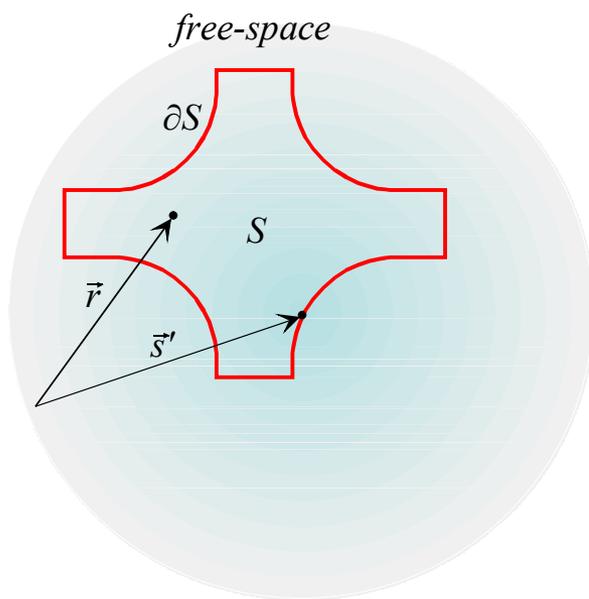
- light mathematical preprocessing
- inhomogeneous medium filling the waveguide
- a single linear eigenvalue problem provides all modes

DISADVANTAGES

- fine surface mesh (**large number of unknowns**)
- spurious solutions

The BEM is based on the **equivalence theorem**: the metallic boundary is replaced by an unknown current density acting in free-space.

The waveguide potentials are represented through **Green's integrals**:



$$\psi(\vec{r}) = \int_{\partial S} g_1^{FS}(\vec{r}, \vec{s}', \kappa) f(\vec{s}') dl' \quad (\text{TM modes})$$

$$\phi(\vec{r}) = \int_{\partial S} g_2^{FS}(\vec{r}, \vec{s}', \kappa) h(\vec{s}') dl' \quad (\text{TE modes})$$

g_1^{FS}, g_2^{FS} **frequency-dependent** Green's functions (involving Henkel functions)

f, h axial and transverse component of the unknown current density

The **integral equation** is obtained by enforcing the boundary conditions, and it is solved by using the **Method of Moments** (MoM).

The resulting matrix equation is in the form:

$$\mathbf{A}(\kappa) \mathbf{X} = \mathbf{0}$$

where matrix \mathbf{A} is frequency-dependent. The problem is solved by determining the **zeros of the determinant of matrix \mathbf{A}**

$$\det\{\mathbf{A}(\kappa)\} = \mathbf{0}$$

by using an iterative method, which is usually not reliable and very time-consuming.



ADVANTAGES

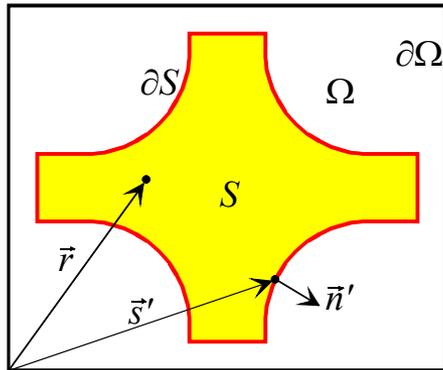
- accurate representation of the waveguide shape
- **one-dimensional grid** on the waveguide boundary

DISADVANTAGES

- homogeneous medium filling S
- moderate mathematical preprocessing
- **frequency-by-frequency calculation**
- possible missing of some modes

- Classical methods for calculating arbitrarily-shaped waveguide modes
- Calculation of **arbitrarily-shaped waveguide** (and cavity) **modes** by the **Boundary Integral-Resonant Mode Expansion (BI-RME) method**
- BI-RME modeling of planar and 3D waveguide circuits
- Fast optimization of waveguide components
- Conclusions

exterior resonator



- The unknown current density acts within an *exterior resonator* with cross-section Ω
- The waveguide potentials are represented in the enlarged domain Ω as follows:

$$\psi(\vec{r}) = \underbrace{\int_{\partial S} G(\vec{r}, \vec{s}') f(\vec{s}') dl'}_{\text{BI}} + \underbrace{\sum_i a_i \Psi_i(\vec{r})}_{\text{RME}} \quad (\text{TM modes})$$

$$\phi(\vec{r}) = \int_{\partial S} \left(\frac{\partial F_0(\vec{r}, \vec{s}')}{\partial n'} + \kappa'' \frac{\partial F_1(\vec{r}, \vec{s}')}{\partial n'} \right) h(\vec{s}') dl' + \kappa''^2 \sum_i b_i \Phi_i(\vec{r}) \quad (\text{TE modes})$$

where: G, F_1, F_2 are static Green's functions of the exterior resonator

Ψ, Φ are TM and TE potentials of the exterior resonator

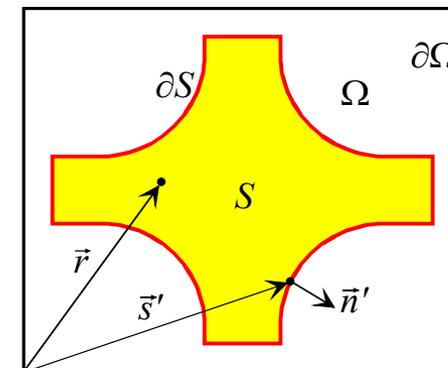
f, h are axial and transverse component of the current density

$$\psi(\vec{r}) = \underbrace{\int_{\partial S} G(\vec{r}, \vec{s}') f(\vec{s}') dl'}_{\psi_0} + \underbrace{\sum_i a_i \Psi_i(\vec{r})}_{\tilde{\psi}}$$

$$\nabla_T^2 \psi + \kappa'^2 \psi = 0 \quad \text{in } \Omega$$

$$\psi = 0 \quad \text{on } \partial\Omega$$

$$\left. \frac{\partial \psi}{\partial n} \right|_{out} - \left. \frac{\partial \psi}{\partial n} \right|_{in} = f \quad \text{on } \partial S$$



For particular values of κ' a function f can be found such that

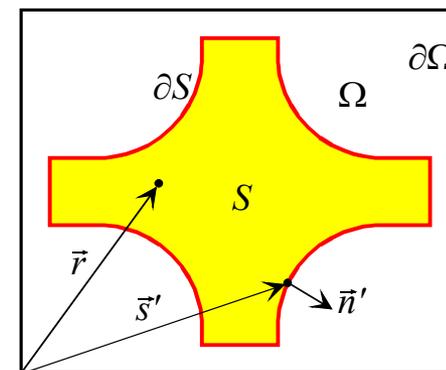
$$\psi = 0 \quad \text{also on } \partial S$$

These values are the resonant wavenumbers of the internal region S or of the external region $\Omega-S$

Functions ψ_0 and $\tilde{\psi}$ satisfy

$$\begin{aligned}\nabla_T^2 \psi_0 &= 0 && \text{in } \Omega - \partial S \\ \psi_0 &= 0 && \text{on } \partial\Omega \\ \left. \frac{\partial \psi_0}{\partial n} \right|_{out} - \left. \frac{\partial \psi_0}{\partial n} \right|_{in} &= f && \text{on } \partial S\end{aligned}$$

$$\begin{aligned}\nabla_T^2 \tilde{\psi} + \kappa'^2 \tilde{\psi} &= -\kappa'^2 \psi_0 && \text{in } \Omega \\ \tilde{\psi} &= 0 && \text{on } \partial\Omega\end{aligned}$$



By imposing the required boundary condition:

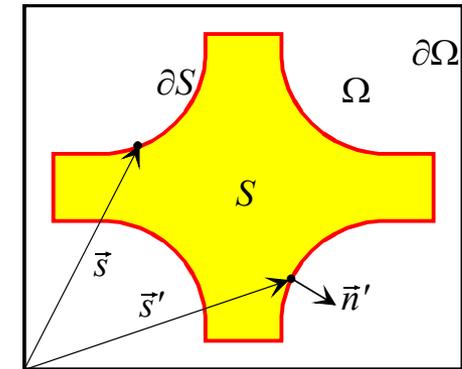
$$\psi = \psi_0 + \tilde{\psi} = 0 \quad \text{on } \partial S$$



$$\tilde{\psi} = -\psi_0 \quad \text{on } \partial S$$

Since

$$\tilde{\psi}(\vec{s}) = -\psi_0(\vec{s}) = - \underbrace{\int_{\partial S} G(\vec{s}, \vec{s}') f(\vec{s}') dl'}_{L\{f\}}$$

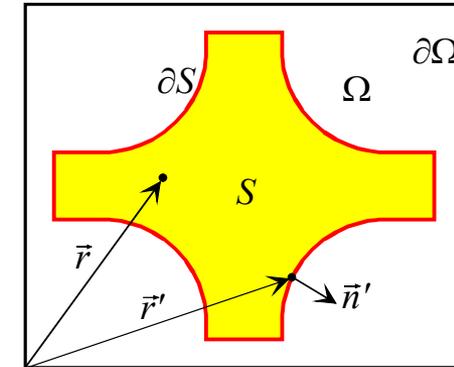


by inverting the integral operator L we obtain

$$\boxed{f = -L^{-1} \{\tilde{\psi}\}} \quad \Rightarrow \quad \boxed{\psi_0(\vec{r}) = - \int_{\partial S} G(\vec{r}, \vec{s}') \left(L^{-1} \{\tilde{\psi}\} \right)_{s'} dl'}$$

$$\psi_0(\vec{r}) = - \int_{\partial S} G(\vec{r}, \vec{s}') \left(\mathbf{L}^{-1} \tilde{\psi} \right)_{s'} dl'$$

$$\begin{aligned} \nabla_T^2 \tilde{\psi} + \kappa'^2 \tilde{\psi} &= -\kappa'^2 \psi_0 && \text{in } \Omega \\ \tilde{\psi} &= 0 && \text{on } \partial\Omega \end{aligned}$$



$$\nabla_T^2 \tilde{\psi}(\vec{r}) + \kappa'^2 \left(\tilde{\psi}(\vec{r}) - \int_{\partial S} G(\vec{r}, \vec{s}') \left(\mathbf{L}^{-1} \tilde{\psi} \right)_{s'} dl' \right) = 0 \quad \text{in } \Omega$$

$$\tilde{\psi}(\vec{r}) = 0 \quad \text{on } \partial\Omega$$

The BI-RME method consists of two steps:

1 – numerical inversion of the operator L , i.e. solution of the deterministic **Boundary Integral** equation:

$$\psi_0(\vec{r}) = - \int_{\partial S} G(\vec{r}, \vec{s}') \left(L^{-1} \tilde{\psi} \right)_{s'} dl'$$

2 – numerical solution of the eigenvalue equation

$$\begin{aligned} \nabla_T^2 \tilde{\psi} + \kappa'^2 \left(\tilde{\psi} - \int_{\partial S} G(\vec{r}, \vec{s}') \left(L^{-1} \tilde{\psi} \right)_{s'} dl' \right) &= 0 & \text{in } \Omega \\ \tilde{\psi} &= 0 & \text{on } \partial\Omega \end{aligned}$$

where $\tilde{\psi}$ is represented by a **Resonant Mode Expansion** involving the potential of the TM resonant modes of the exterior resonator Ω

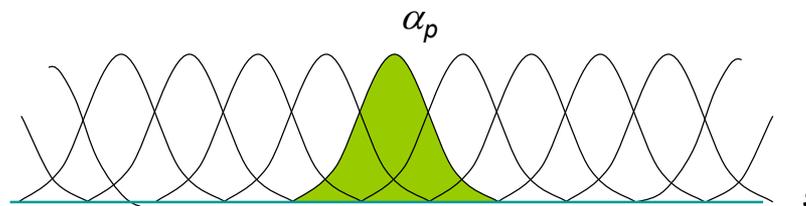


Both problems are solved by using the MoM

- Expansion of f

$$f(\vec{s}') = \sum_p c_p \vec{\alpha}_p(\vec{s}') = \mathbf{U}^T \mathbf{c}$$

$$\mathbf{U} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_P \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_P \end{bmatrix}$$



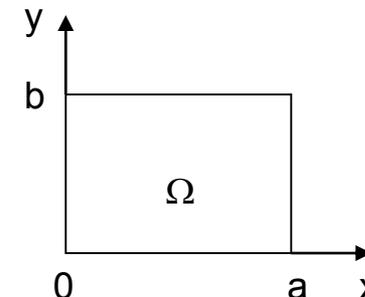
- Expansion of $\tilde{\psi}$

$$\tilde{\psi}(\vec{r}) = \sum_j a_j \Psi_j(\vec{r}) = \mathbf{\Psi}^T \mathbf{a}$$

$$\mathbf{\Psi} = \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_J \end{bmatrix} \quad \mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_J \end{bmatrix}$$

$$\Psi_j = \sqrt{\frac{2}{ab}} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

$$\lambda_j = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$



Indicating with (\cdot, \cdot) the inner product in $L_2(\partial S)$, by the Galerkin method we obtain

$$f = -\mathbf{L}^{-1} \tilde{\psi} = \mathbf{U}^T \mathbf{L}^{-1} (\mathbf{U}, \Psi^T \mathbf{a})$$

$\mathbf{L} = (\mathbf{U}, \mathbf{L}\mathbf{U}^T)$ $P \times P$ symmetric,
positive defined real matrix

$$\nabla_T^2 \tilde{\psi} + \kappa'^2 \left(\tilde{\psi} - \int_{\partial S} G(\vec{r}, \vec{s}') (\mathbf{L}^{-1} \tilde{\psi})_{s'} dl' \right) = 0$$

$$\Lambda^2 \mathbf{a} = \kappa'^2 (\mathbf{1} - \Lambda^{-2} \mathbf{R}^T \mathbf{L}^{-1} \mathbf{R}) \mathbf{a}$$

$$\Lambda = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_J\}$$

$$\mathbf{R} = (\mathbf{U}, \Psi^T) \quad P \times J \text{ real matrix}$$

LINEAR
EIGENVALUE
PROBLEM

- The solution of a linear matrix eigenvalue problem permits to find all TM modes in a prescribed frequency band
- A similar procedure has been developed for TE modes
- The potential in S can be calculated by

$$\psi_i(\vec{r}) = - \left(\int_{\partial S} G(\vec{r}, \vec{s}') \mathbf{U}^T(\vec{s}') dl' \right) \mathbf{L}^{-1} \mathbf{R} \mathbf{a}_i + \mathbf{\Psi}^T(\vec{r}) \mathbf{a}_i$$

- The derivative of the potential on ∂S is a by-product of the method

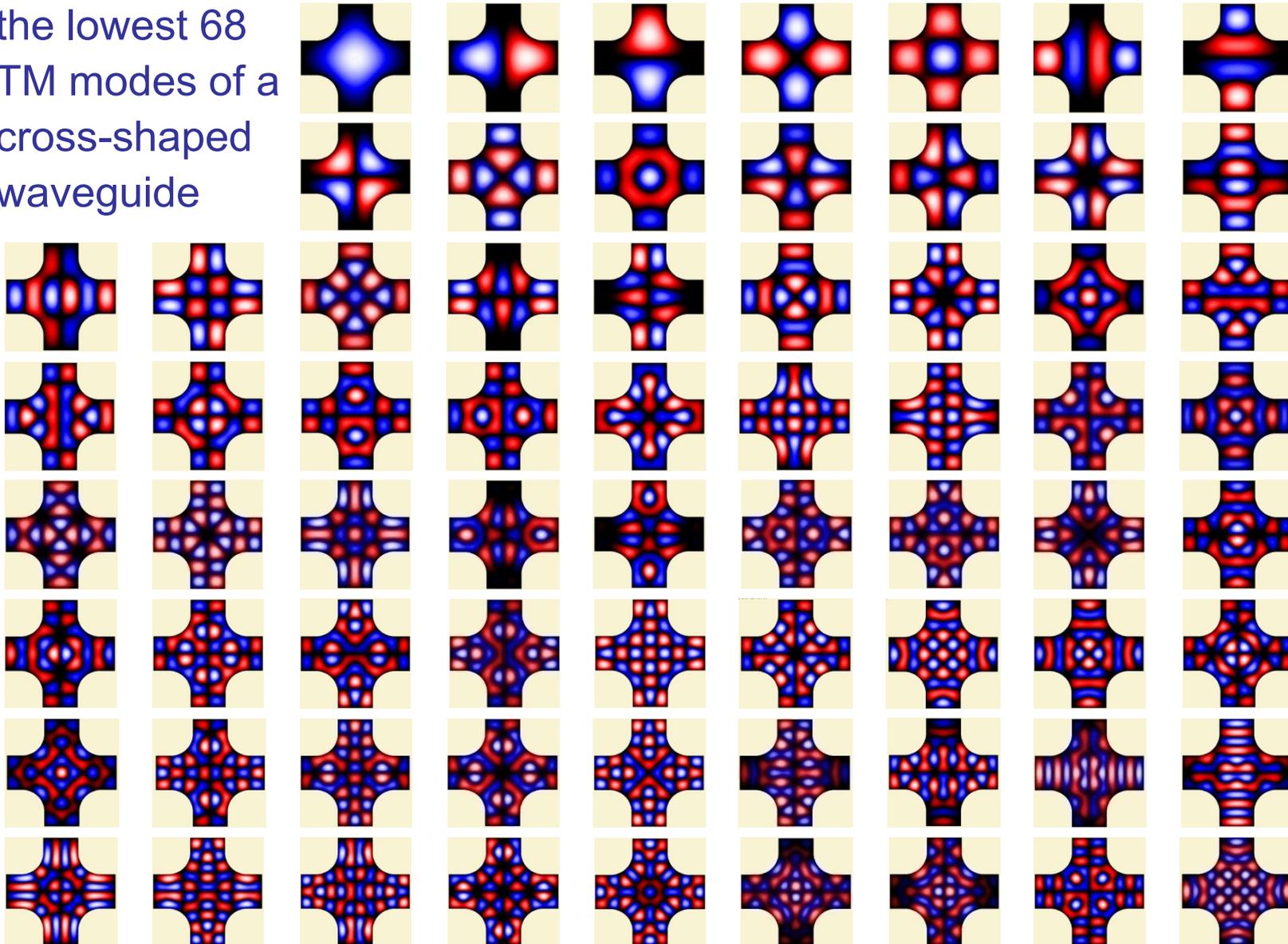
$$\frac{\partial \psi_i}{\partial n} = -f = -\mathbf{U}^T \mathbf{L}^{-1} (\mathbf{U}, \mathbf{\Psi}^T \mathbf{a}_i)$$

(all the matrices are calculated during the analysis)

BI-RME METHOD: EXAMPLE

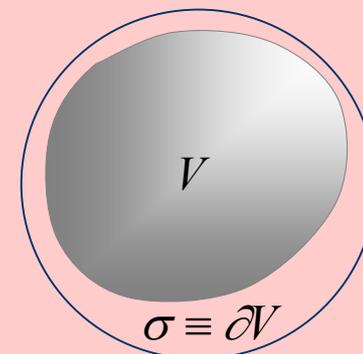
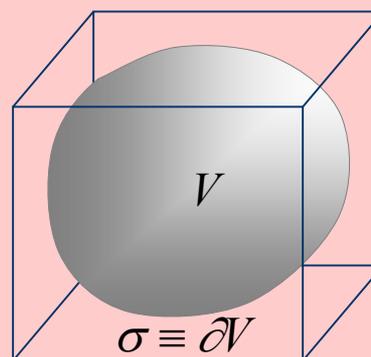


the lowest 68
TM modes of a
cross-shaped
waveguide



The BI-RME method can also be used for determining the eigenfunctions of the 3D vector wave equation with electric-wall boundary condition

$$\begin{aligned} \nabla \times \nabla \times \vec{E} + k^2 \vec{E} &= 0 & \text{on } V \\ \vec{n} \times \vec{E} &= 0 & \text{on } \partial V \end{aligned}$$



SAME PHILOSOPHY, SAME ADVANTAGES ...

... MORE COMPLICATED:

- vector BI-RME representation
- vector resonant modes
- dyadic Greens functions

ADVANTAGES

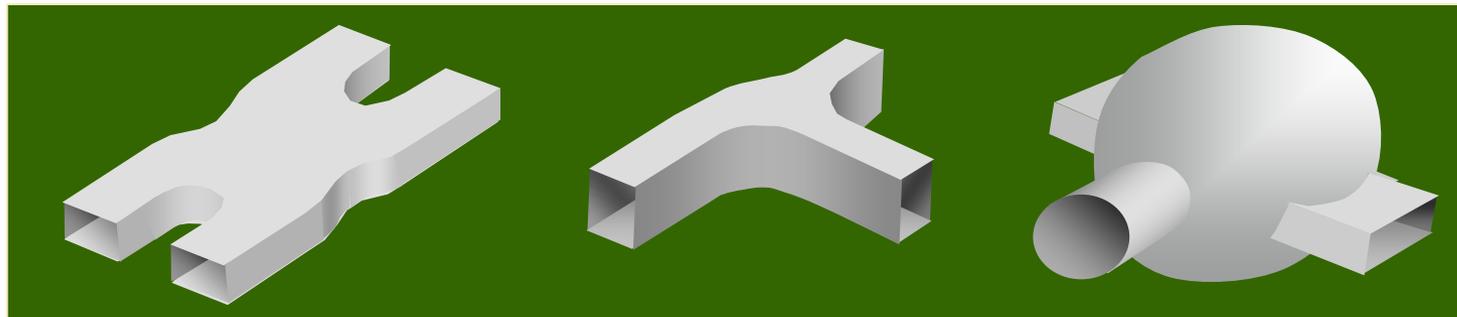
- accurate representation of the waveguide shape
- one-dimensional grid on the waveguide boundary
- solution of a single linear eigenvalue problem
- the modal fields are automatically normalized

DISADVANTAGES

- homogeneous medium filling S
- heavy mathematical preprocessing



- Classical methods for calculating arbitrarily-shaped waveguide modes
- Calculation of arbitrarily-shaped waveguide (and cavity) modes by the Boundary Integral-Resonant Mode Expansion (BI-RME) method
- **BI-RME modeling of planar and 3D waveguide circuits**
- Fast optimization of waveguide components
- Conclusions



H-plane circuit

E-plane circuit

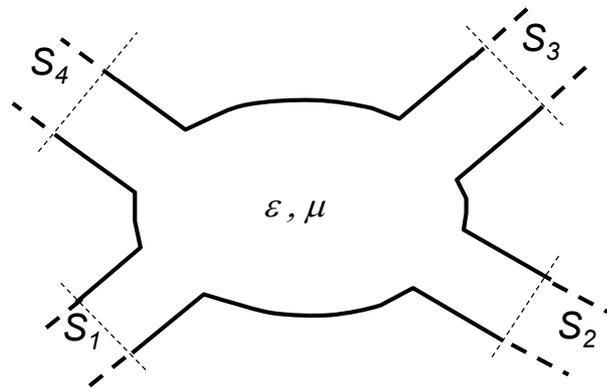
3D circuit

The BI-RME method yields the **mathematical model** of a waveguide component in the form of the pole expansion of the Y-parameters:

$$Y_{ij} = \frac{1}{j\omega} A_{ij} + j\omega B_{ij} + j\omega^3 \sum_{m=1}^M \frac{C_{im} C_{jm}}{\omega_m^2 (\omega_m^2 - \omega^2)}$$

low-frequency terms

modes of the cavity



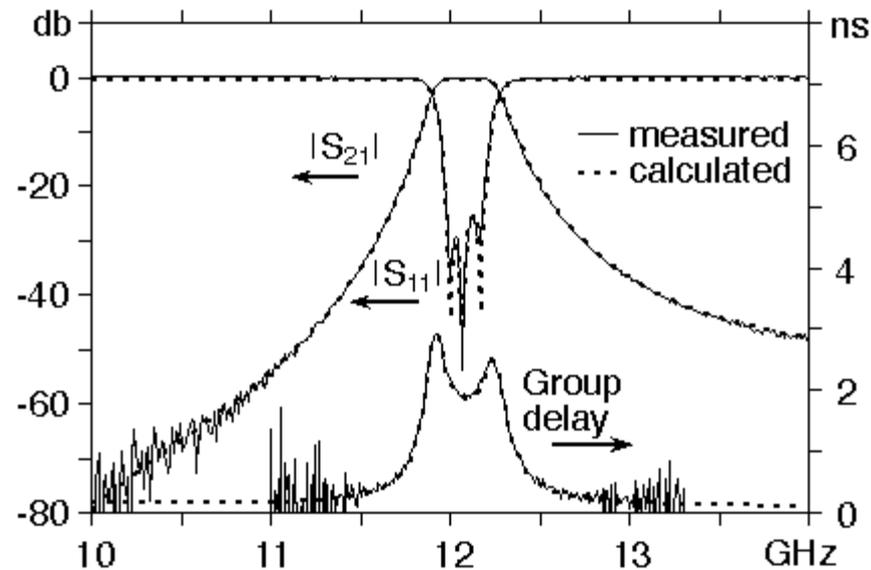
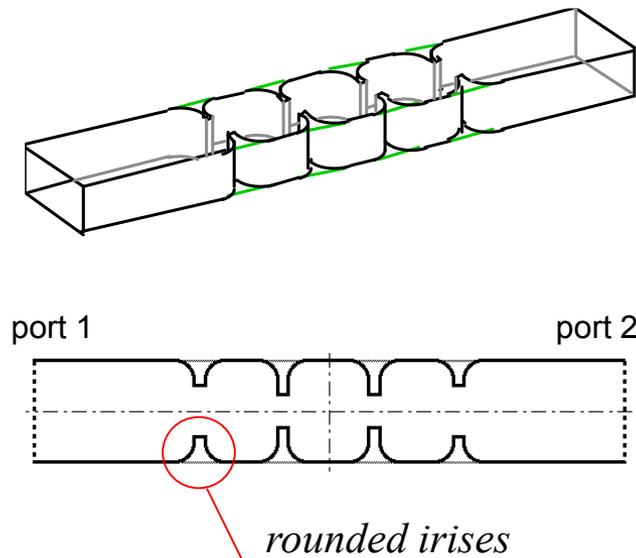
$$Y_{ij} = \underbrace{\frac{1}{j\omega} A_{ij} + j\omega B_{ij}}_{\text{low-frequency terms}} + j\omega^3 \underbrace{\sum_{m=1}^M \frac{C_{im} C_{jm}}{\omega_m^2 (\omega_m^2 - \omega^2)}}_{\text{modes of the cavity}}$$

Very **simple analytical expressions** yield the A - and B -coefficients in terms of the cutoff wavelengths of the terminal waveguides (provided they are sufficiently long)

Constants ω_m and C_{im} are related to the **resonant modes** of the component with the ports closed by electric walls. These modes are determined by the BI-RME method.

H-plane three-cavity filter

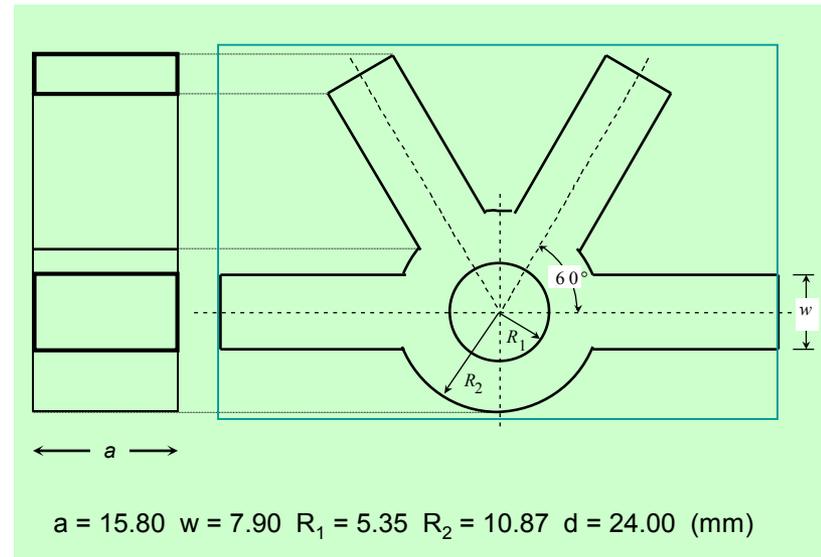
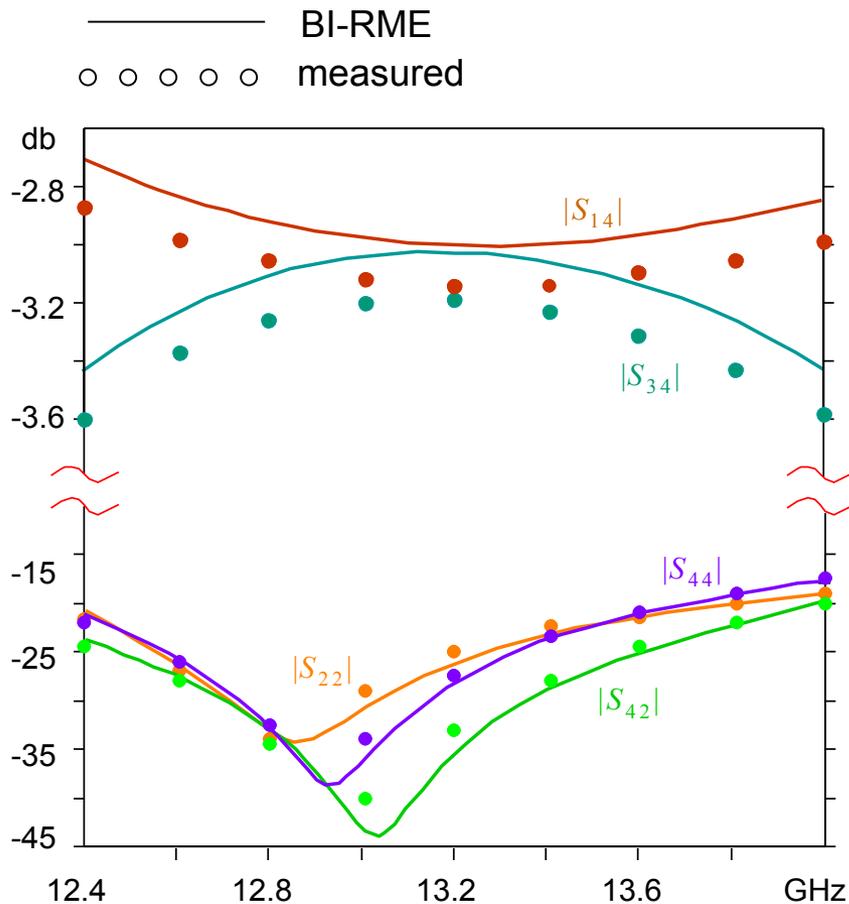
*modelling performed by using
ANAPLAN-W*



- CPU time: 1.3 sec (Sun Ultra 10) (accuracy factor: $\zeta = 4.5$)
- measurements refer to a prototype machined with very tight tolerances at ESA-ESTEC

E-plane directional coupler

modelling performed by using ANAPLAN-W

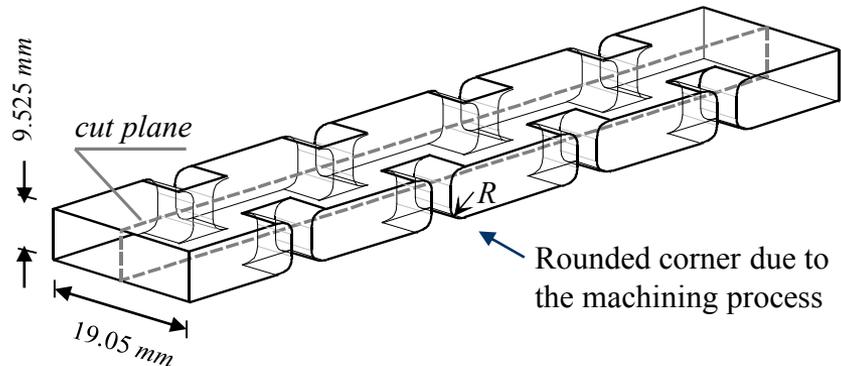


CPU time: 20 sec

(accuracy factor $\zeta = 3.5$)

longer than in the previous cases because almost no part of the boundary fits with the rectangular boundary

THREE-CAVITY WAVEGUIDE FILTER



3D structure to be embedded in a spherical or rectangular domain Ω

Spherical case:

- no part of the boundary matches with the sphere (the surface σ coincides with ∂V).
- a big sphere must be considered to include long terminal waveguides

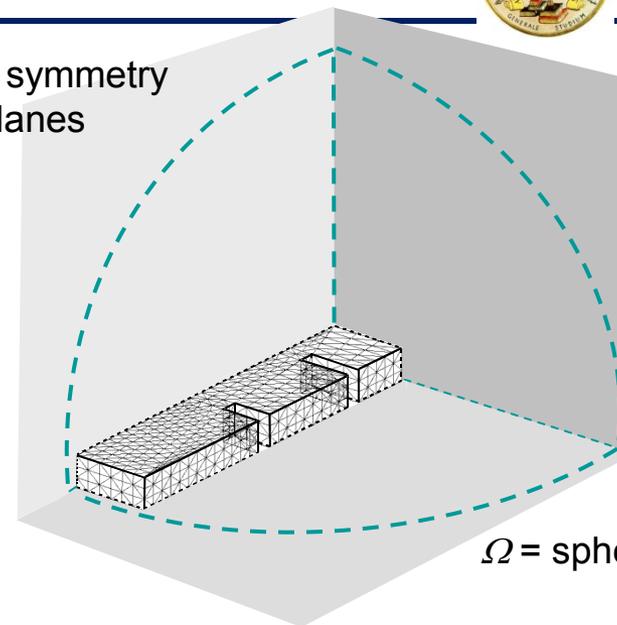
Large number of variables both in the BI and in the RME

Rectangular case:

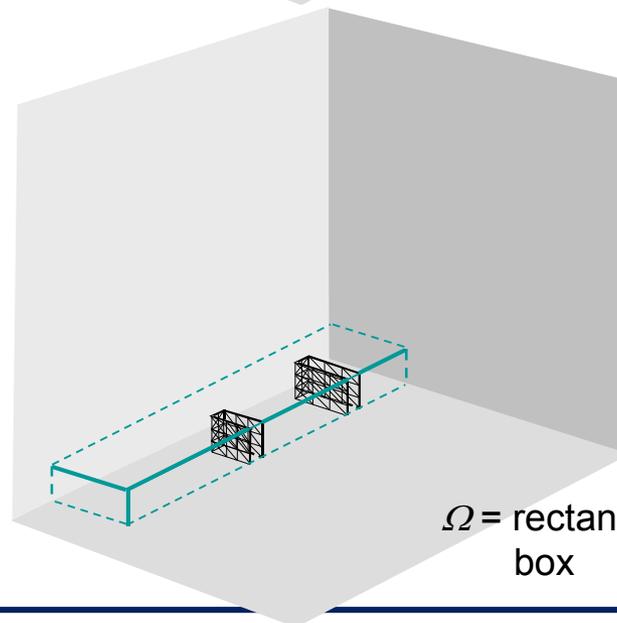
- most of the boundary matches with the box (the surface σ reduces to the irises)
- volume Ω much smaller

Number of variables much reduced

3 symmetry
planes

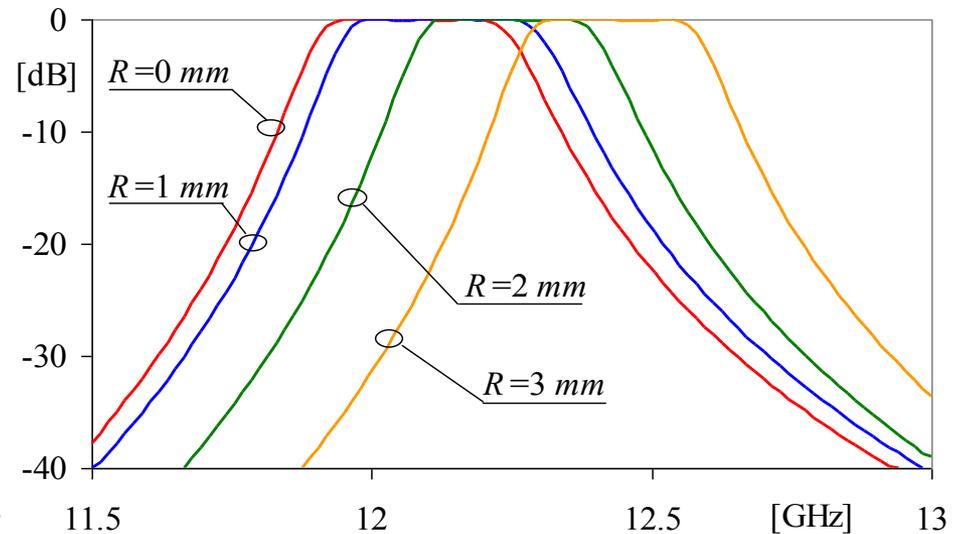
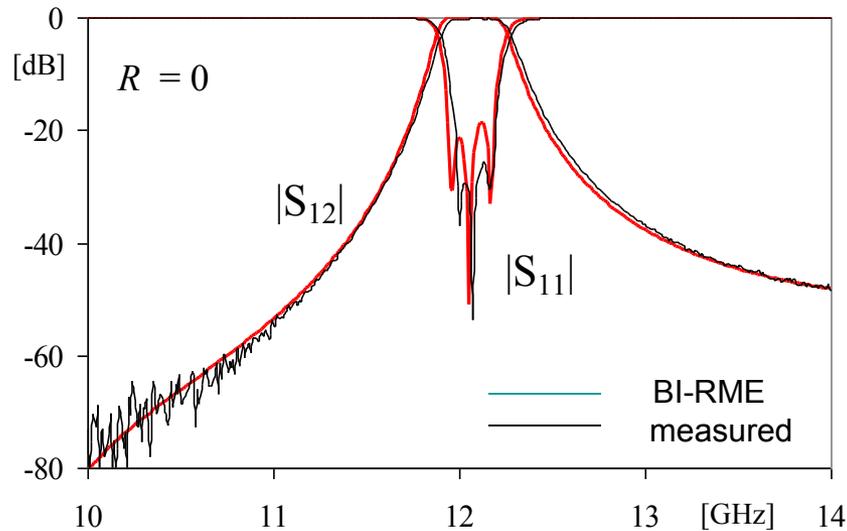
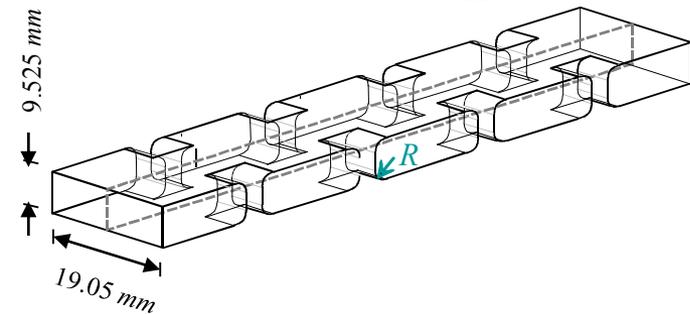


$\Omega = \text{sphere}$



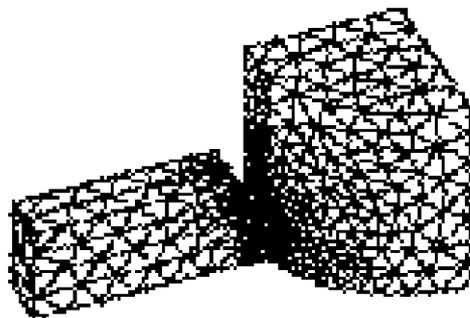
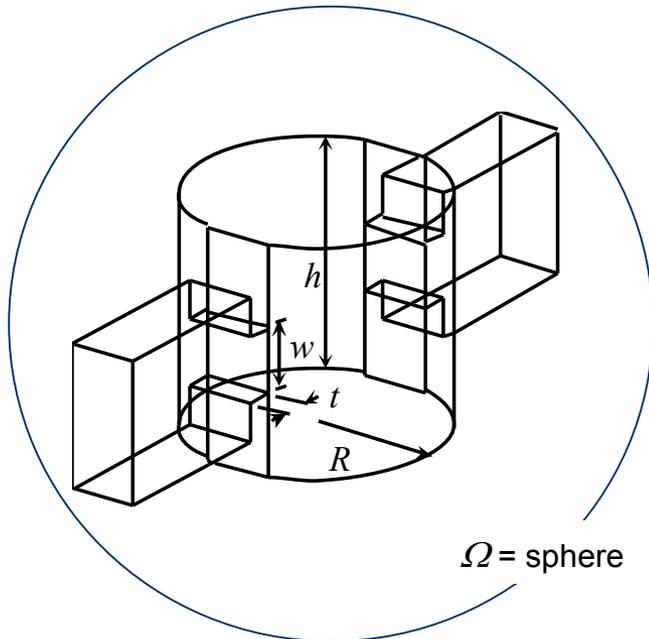
$\Omega = \text{rectangular box}$

THREE-CAVITY WAVEGUIDE FILTER

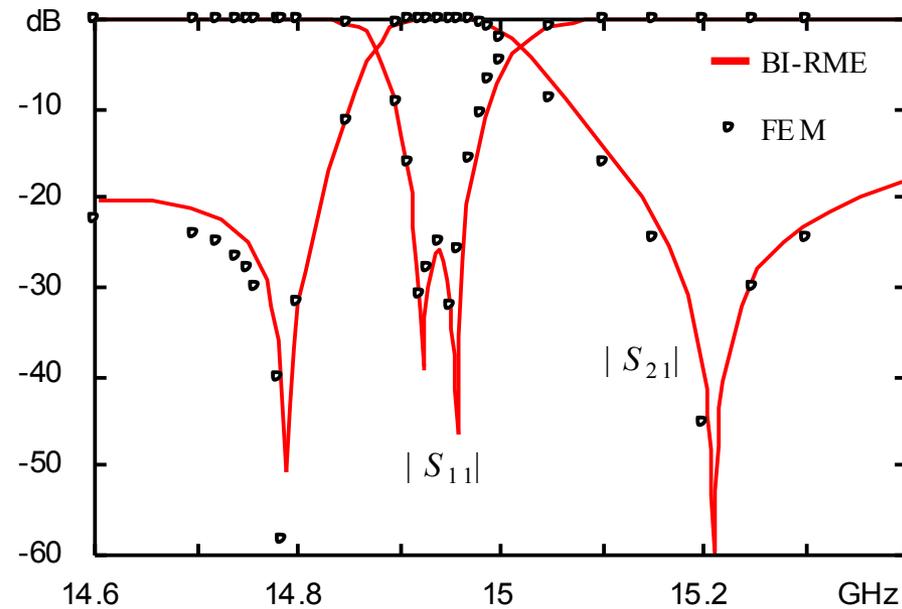


external resonator	basis functions (Q)	resonant modes (P)	filling of BI matrices	filling of RME matrices	eigenvalue calculation	calculation of \mathbf{C}	total
boxed	166	50	90 sec	1 sec	1 sec	42 sec	134 sec
spherical	1087	380	295 sec	60 sec	180 sec	22 sec	557 sec

3D DUAL-MODE FILTER



mesh (symmetry exploited)



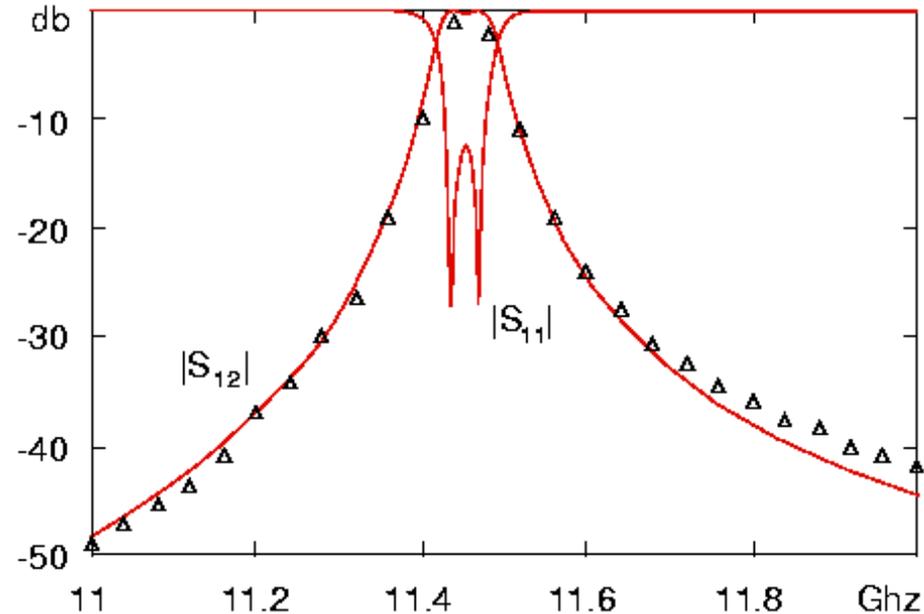
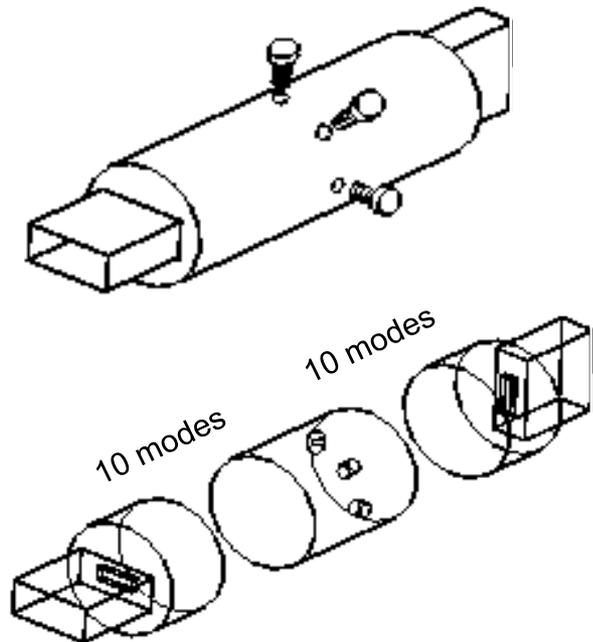
BI-RME analysis (solid lines) compared with HFSS (markers)

Computing time:

BI-RME 20 min on a Digital Alpha 200^{4/233}
(accuracy factor $\xi = 3.5$)

FEM hours

DUAL MODE CIRCULAR WAVEGUIDE FILTER



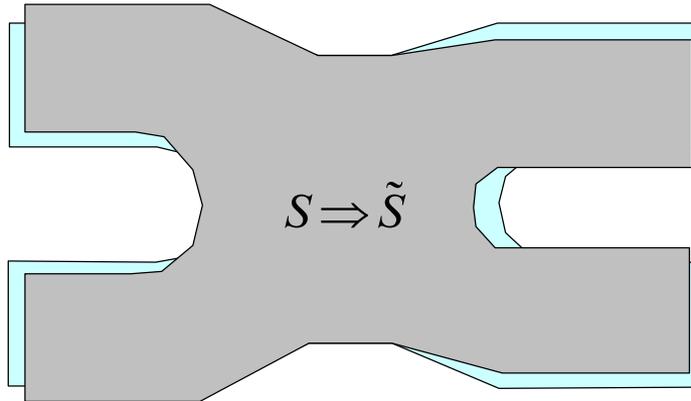
BI-RME analysis (solid line) compared with measurements (markers)

- computing time: 15 min on a Digital Alpha 200^{4/233}
- screws simulated as cylinders



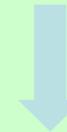
- Classical methods for calculating arbitrarily-shaped waveguide modes
- Calculation of arbitrarily-shaped waveguide (and cavity) modes by the Boundary Integral-Resonant Mode Expansion (BI-RME) method
- BI-RME modeling of planar and 3D waveguide circuits
- **Fast optimization** of waveguide components
- Conclusions

Both the tolerance analysis and the optimization require to evaluate the effect of the perturbation of S on the quantities k_i, C_{pi}



A_{pq}, B_{pq} depend on the width and the height of the terminal waveguides (not involved in the optimization)

$$Y_{pq} = \frac{A_{pq}}{jk\eta} + \frac{jk}{\eta} B_{pq} + j \frac{k^3}{\eta} \sum_i \frac{C_{pi} C_{qi}}{k_i^2 - k^2}$$



$$\tilde{Y}_{pq} = \frac{A_{pq}}{jk\eta} + \frac{jk}{\eta} B_{pq} + j \frac{k^3}{\eta} \sum_i \frac{\tilde{C}_{pi} \tilde{C}_{qi}}{\tilde{k}_i^2 - k^2}$$

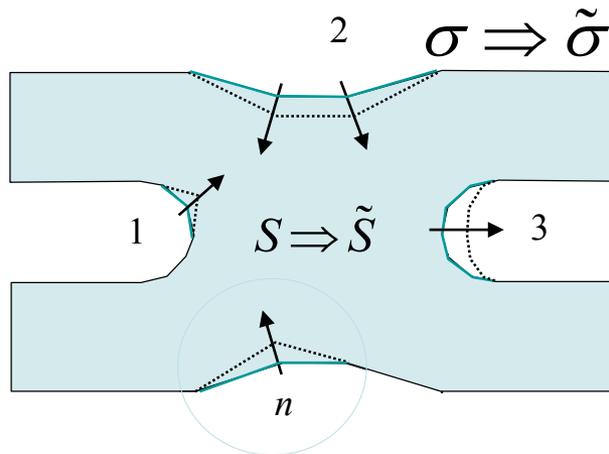
The perturbed poles and residues depend on the perturbed eigensolutions of the Helmholtz equation

Boundary conditions:

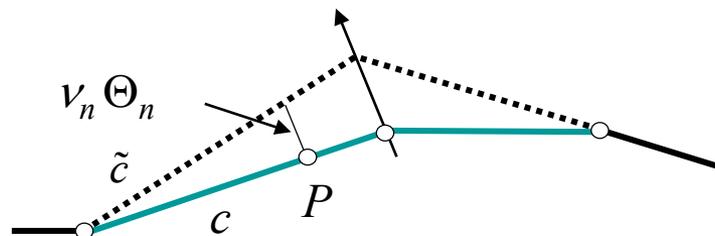
- Dirichelet (H-plane components)
- Neumann (E-plane components)

$$\kappa_i \Rightarrow \tilde{\kappa}_i \quad \psi_i \Rightarrow \tilde{\psi}_i$$

Shape perturbations



n -th perturbation



v_n = perturbation parameter

$\Theta_n = \Theta_n(P)$ = shape function

$$\tilde{\kappa}^2 = \frac{\int_{\tilde{S}} \nabla \tilde{\psi} \cdot \nabla \tilde{\psi} \, d\tilde{S} - 2\beta \int_{\tilde{c}} \tilde{\psi} \, \partial \tilde{\psi} / \partial n \, d\tilde{c}}{\int_{\tilde{S}} \tilde{\psi}^2 \, d\tilde{S}}$$

$\beta = 1$ for Neumann b.c.

$\beta = 0$ for Dirichlet b.c.

Trial function:

$$\tilde{\psi} = \sum_{j=1}^M x_j \psi_j$$

By applying the **Rayleigh-Ritz** method the perturbed eigenvalues and eigenfunctions are found by solving a matrix eigenvalue equation. (small order matrices).

Matrix entries are integrals over the perturbed boundary.

The functions to be integrated only involve the values of the eigenfunctions (or of their normal derivative) over the line σ
(primary results of the BI-RME calculation of the unperturbed eigensolution)

optimization procedure

- definition of an optimization goal (in terms of S-parameters) and of a distance R between the actual result and the goal;
- definition of N allowed deformations of the boundary (shape functions $\Theta_1, \Theta_2, \dots, \Theta_N$);
- use of the Rayleigh-Ritz method for determining the perturbed poles and the residues with respect to each one of some given perturbation parameters $\nu_1, \nu_2, \dots, \nu_N$;
- for any ν_n , calculation of the perturbed scattering parameters and of the corresponding distance R ;
- determination of the sensitivities of the distance R with respect to ν -parameters;
- use of the sensitivities in a quasi-Newton optimization algorithm, to minimize R .

optimization steps are very fast

- the recalculation of the poles and the residues of the Y-matrix is very fast due to the efficiency of the BI-RME method;
- the scattering parameters are evaluated very rapidly in all the band of interest, by using the mathematical model;
- the sensitivities $\partial R / \partial \nu_n$ are obtained very easily by solving N small-size eigenvalue problems.

WG COMPONENT OPTIMIZATION



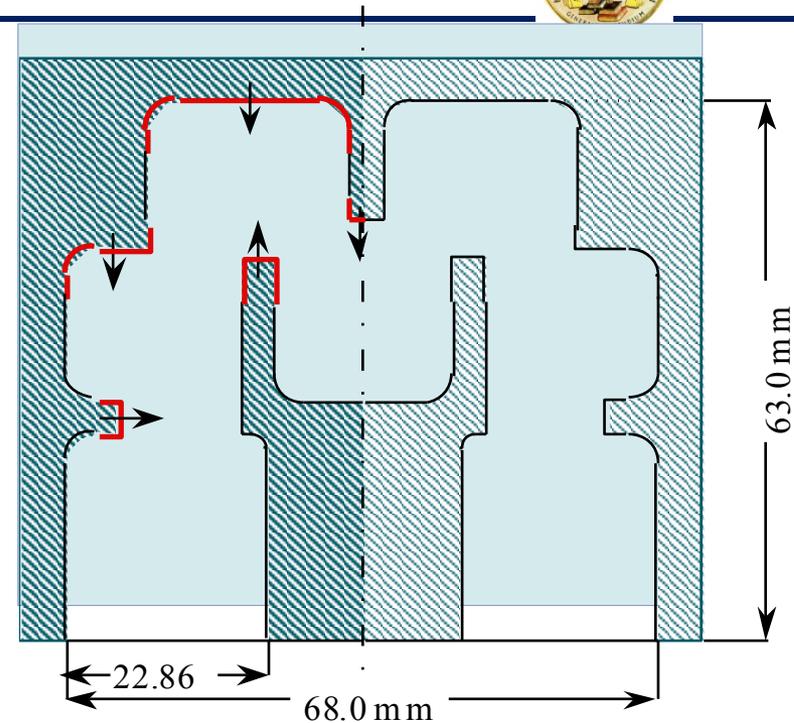
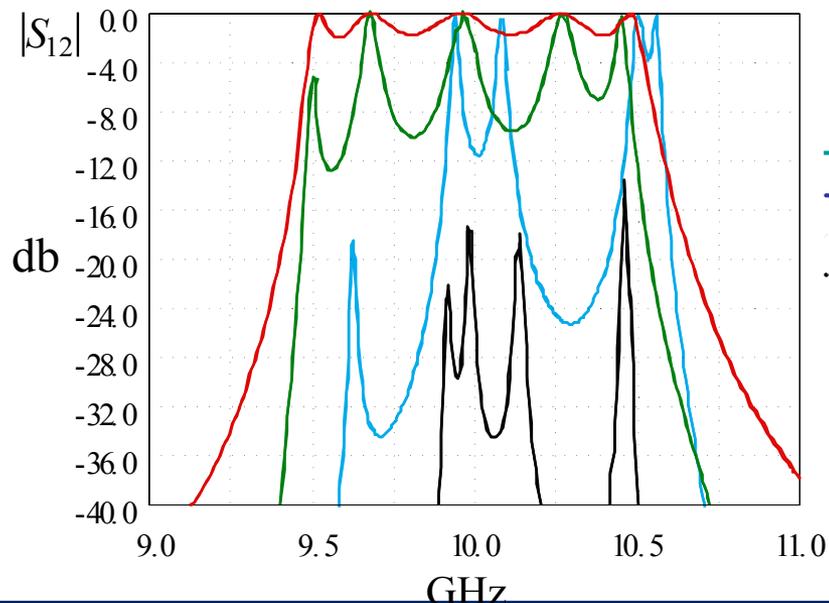
H-plane five-cavity X-band filter

GOALS:

Insertion Loss

< 2 dB in the band 9.5÷10.5 GHz

> 30 dB, $f < 9.3$ GHz, $f > 10.8$ GHz



- after 42 steps
- after 28 steps
- after 14 steps
- initial response

CPU time per step: 3 sec

(Pentium III, 500 MHz, PC)

Symmetry was exploited to speed-up the analysis and simplify the optimization.

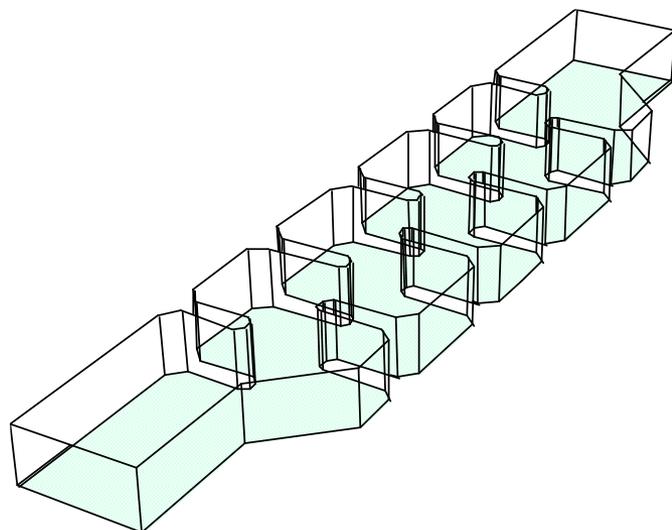
- The “Montecarlo analysis” of the effect of machining inaccuracies is performed by generating a set of many perturbed models of the same component, with random perturbations.
- The perturbations correspond to random sets of the v -parameters, created by a generator of random numbers, according to the given tolerances.

statistical samples consisting of thousands of cases can be generated in short times, due to the rapidity of the calculation of the perturbed poles and residues.

TOLERANCE ANALYSIS

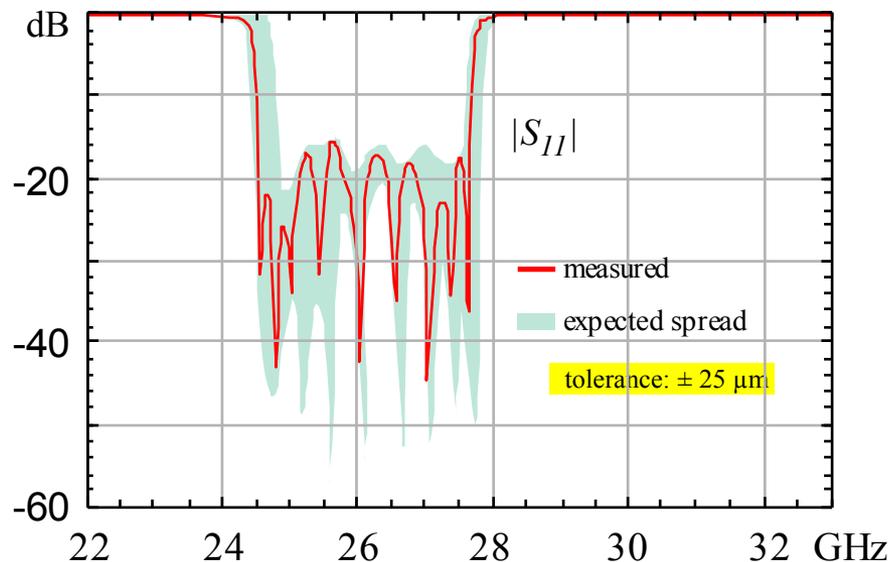
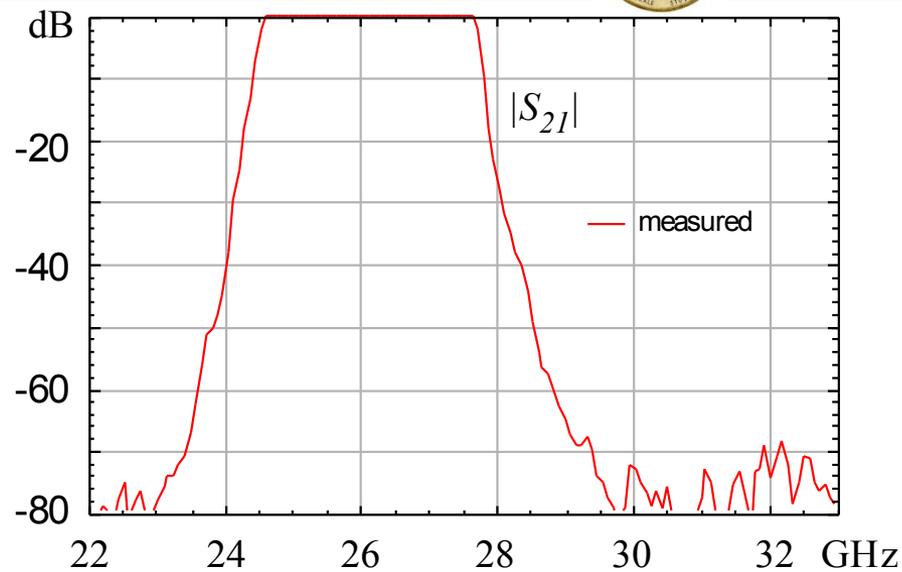


Design and Tolerance Analysis of a Nine Cavity Meander Filter (H-plane)



CPU time per step: 15 sec
(Pentium III, 500 MHz, PC)

Tolerance analysis: 1 min
(statistical sample: 1000 cases)



experimental results and spread area

TOLERANCE ANALYSIS - EXAMPLE



Optimization and tolerance analysis of a WR-22 E-Plane 3-dB Coupler

Goal:

3 ± 0.5 dB coupling in the band 42-48 GHz

30 optimization steps

Total CPU time: 1min (Pentium III, 500 MHz)

Tolerance analysis: 2 min

($\pm 15 \mu\text{m}$ on all the dimensions, 1000 cases)

