Lecture 15

MODELING OF FREQUENCY SELECTIVE SURFACES BY THE MOM/BI-RME METHOD

OUTLINE

- Introduction to frequency selective surfaces (FSS)
- Modeling of infinitely thin capacitive FSS
- Modeling of capacitive FSS with finite thickness
- Modeling of inductive FSS
- Conclusion

OUTLINE

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- Introduction to frequency selective surfaces (FSS)
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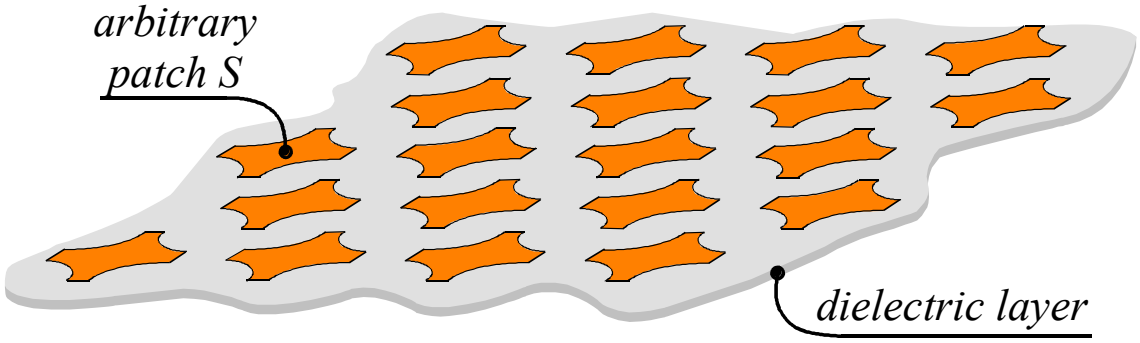
FREQUENCY SELECTIVE SURFACES

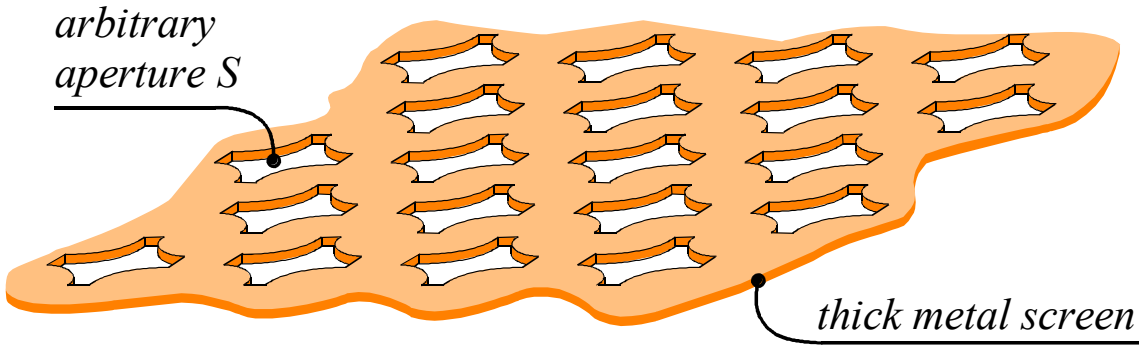


CAPACITIVE FSS

(stopband @ resonance)

INDUCTIVE FSS

(bandpass @ resonance)



APPLICATIONS OF FSS

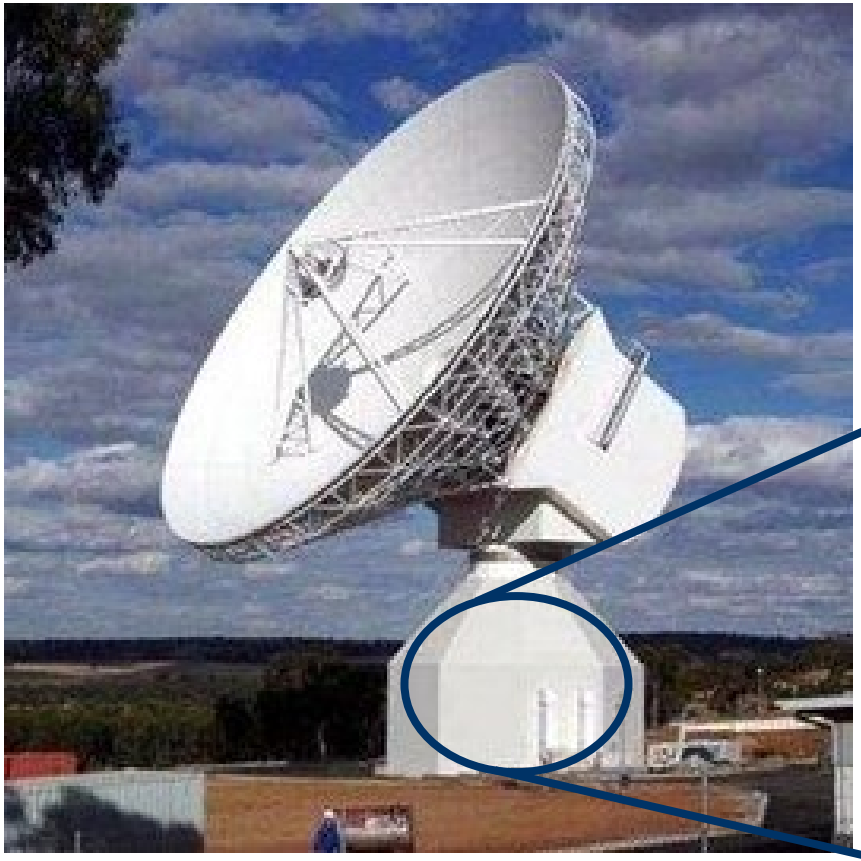
In the MICROWAVE range:

- reflector antennas for radio-astronomy (*beam waveguide antennas*)
- antenna radomes
- consumer electronics (microwave ovens)

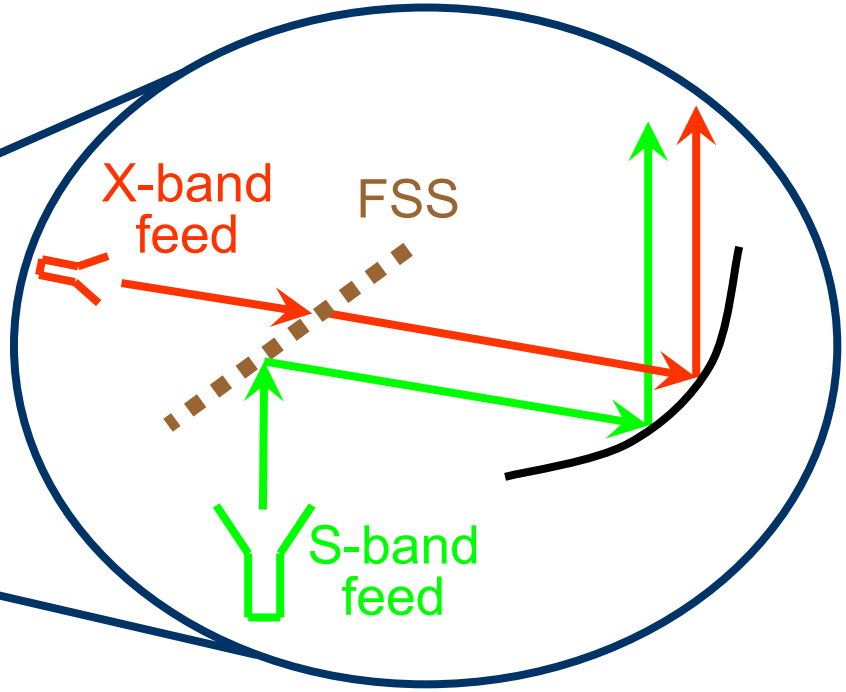
In the MM– and SUBMM–WAVE range:

- quasi–optical systems (*frequency multipliers*)
- laser cavities
- Fabry–Perot interferometers

APPLICATIONS OF FSS

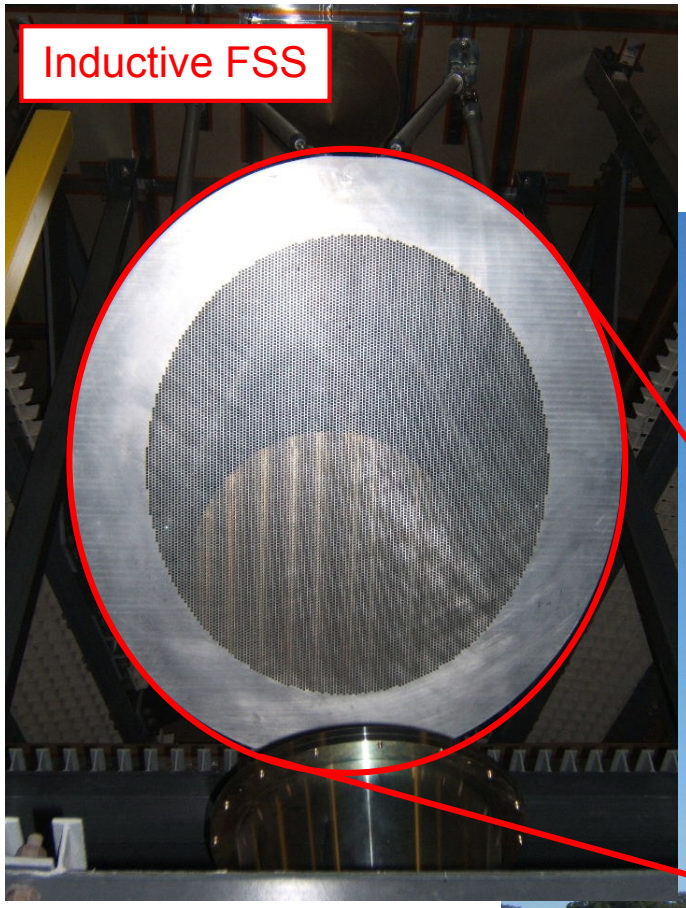


FSS are used in large reflector antennas to combine beams at different frequencies.

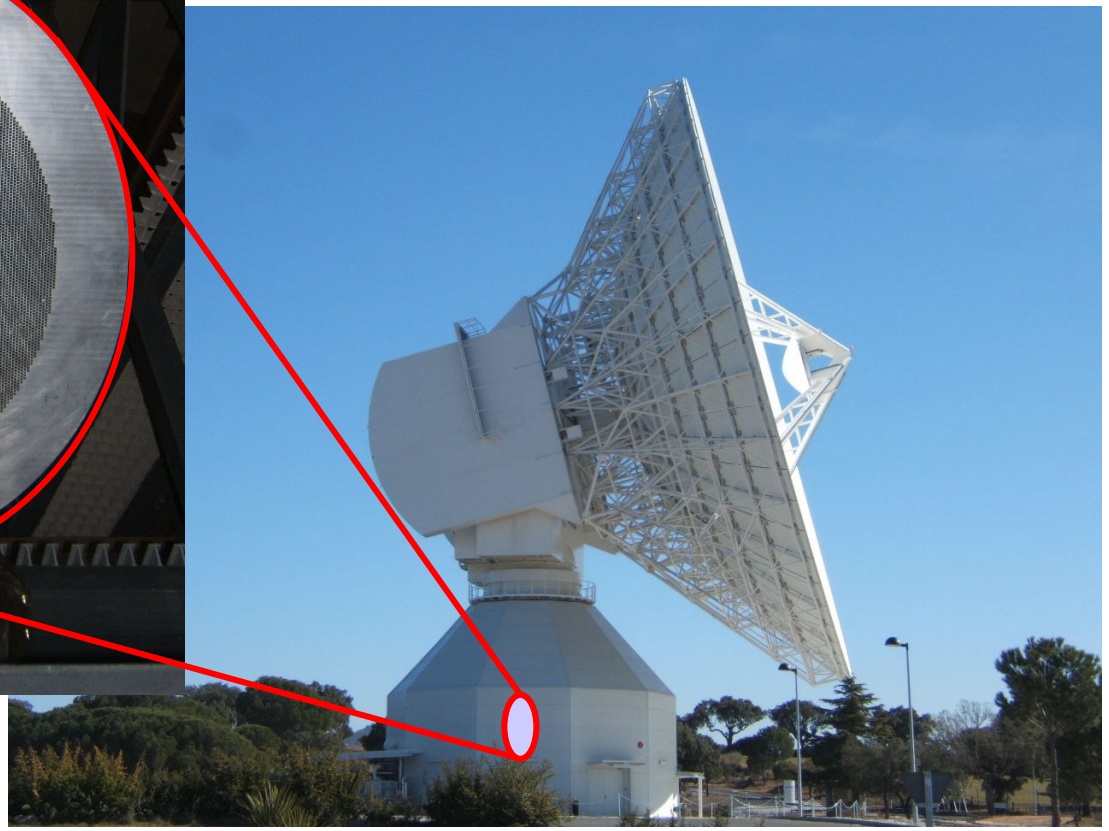


This is ESA's first 35-metre deep-space ground station at New Norcia, 140 km north of Perth in Australia.

APPLICATIONS OF FSS

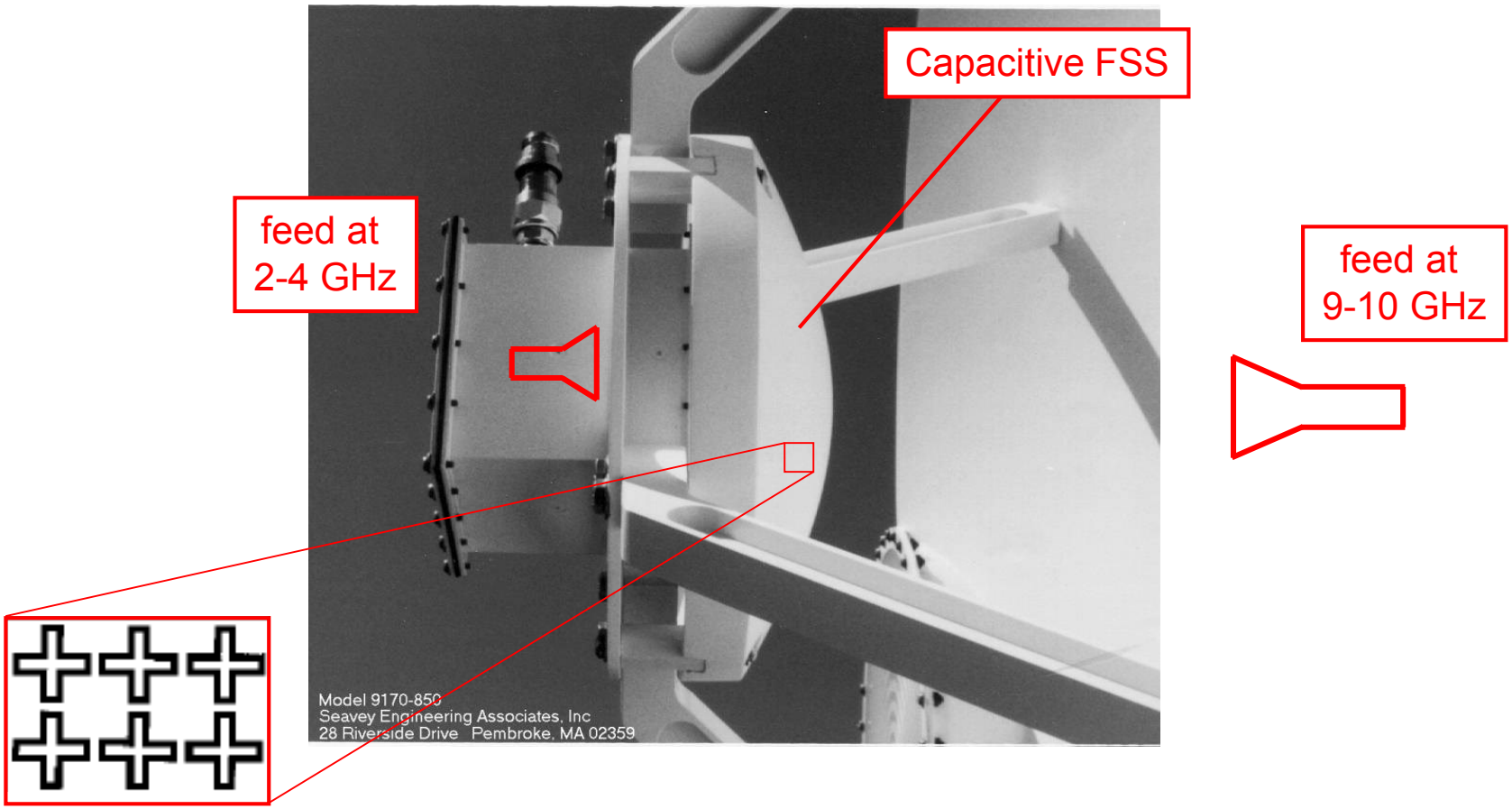


35-meter Deep Space Antenna of ESA in Cebreros, Spain.



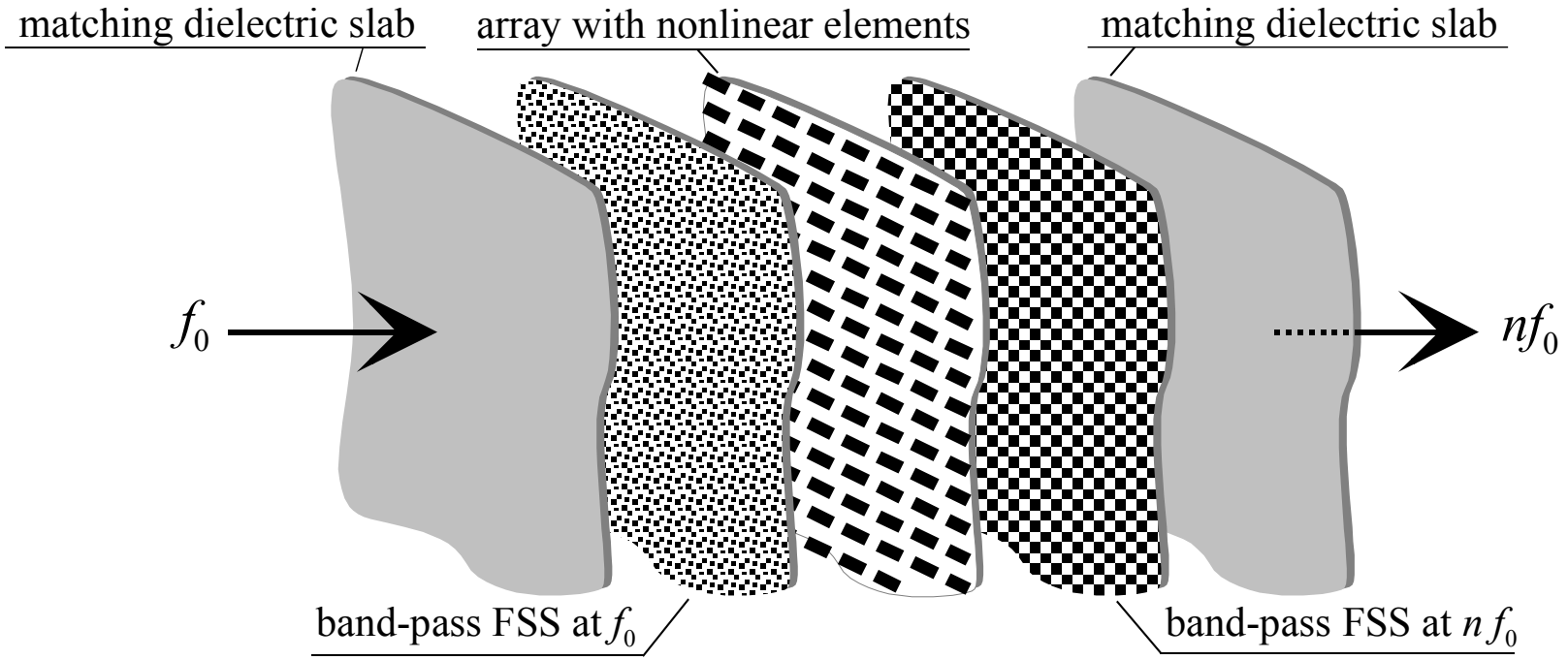
This inductive FSS reflects signals in the 7.145–7.235 GHz and 8.4–8.5 GHz frequency band and is transparent in the 31.8–32.3 GHz and 34.2–34.7 GHz band.

APPLICATIONS OF FSS



This capacitive FSS transmits signals in the 2-4 GHz and reflects signals from 9-10 GHz frequency band. It is formed by squared metal patches printed on a small (8.5-inch diameter) hyperboloid sub-reflector.

APPLICATIONS OF FSS



FSSs are used in quasi-optical frequency multipliers to guarantee unidirectional operation.

OUTLINE

- Introduction to frequency selective surfaces (FSS)
- Modeling of infinitely thin capacitive FSS
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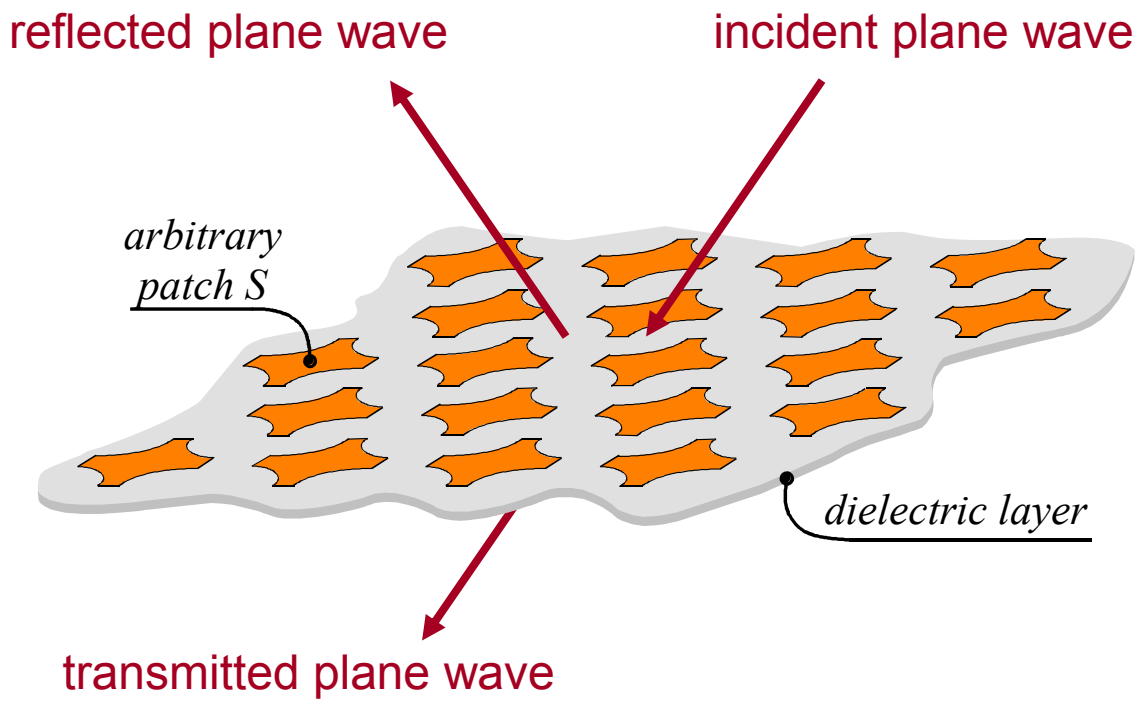
MODELING OF CAPACITIVE FSS



- infinite periodic array of infinitely-thin metal patches
- stratified dielectric medium
- patches with an arbitrary shape
- lossy metal & dielectric

MODELING OF CAPACITIVE FSS

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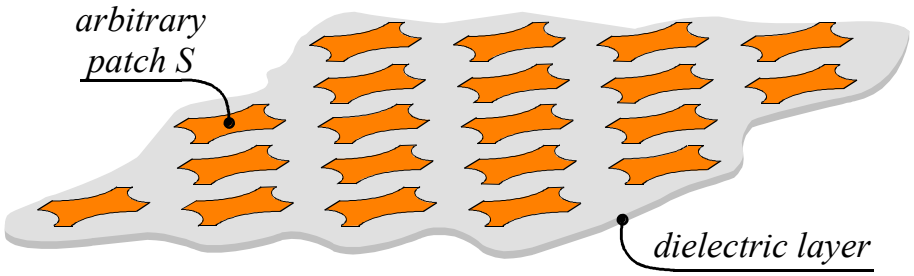
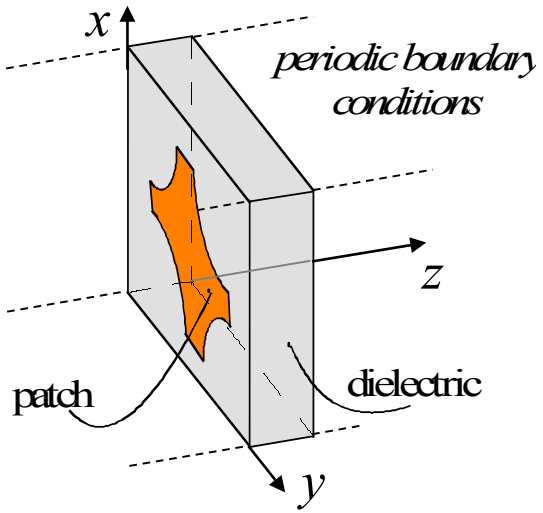
BASIC HYPOTHESES:
- infinite array approximation
- uniform plane wave illumination



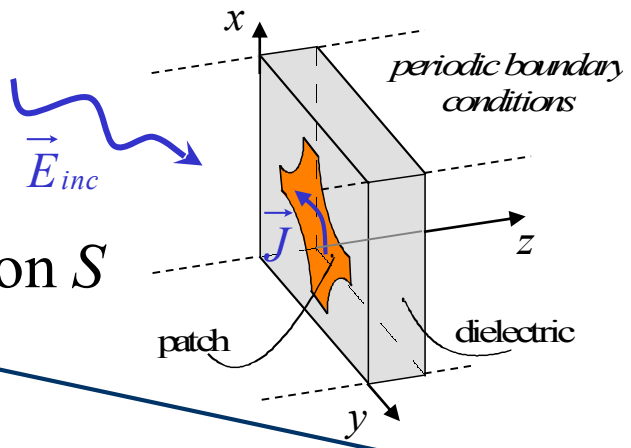


BY FLOQUET THEOREM

- **INTEGRAL EQUATION**

incident field *scattered field* *metal losses*

$$\vec{E}_{inc}(\vec{r}) + \int_S \vec{\vec{G}}(\vec{r},\vec{r}') \cdot \vec{J}(\vec{r}')\, dS' = Z_S \vec{J}(\vec{r}) \quad \text{on } S$$



$$\sum_m V_m(z,z') \quad \vec{\mathcal{E}}_m(x,y)\vec{\mathcal{E}}_m^*(x',y') \qquad \sqrt{\frac{\pi f \mu_0}{\sigma}}(1+j)$$

by the equivalent modal transmission line circuit

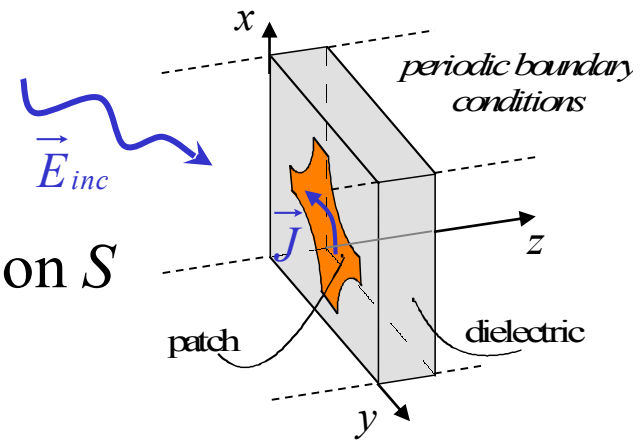
modal vectors of the Floquet modes

surface impedance (skin effect)

THE MOM/BI-RME METHOD / 2

- **INTEGRAL EQUATION**

$$\underbrace{\vec{E}_{inc}(\vec{r})}_{\text{incident field}} + \underbrace{\int_S \vec{\vec{G}}(\vec{r},\vec{r}\,') \cdot \vec{J}(\vec{r}\,')\, dS'}_{\text{scattered field}} = \underbrace{Z_S \vec{J}(\vec{r})}_{\text{metal losses}} \quad \text{on } S$$



- **METHOD OF MOMENTS (MOM)**

$$\vec{J}(\vec{r}) = \sum_{i=1}^{N} X_i \vec{e}_i\,(x',y')$$

entire-domain basis functions (BI-RME method)

- **RESULTING MATRIX PROBLEM**

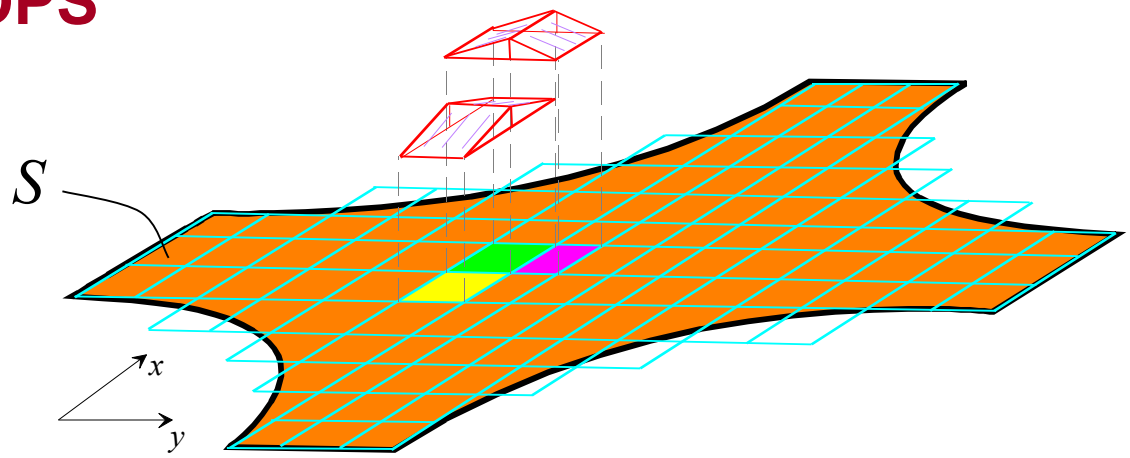
$$\mathbf{A}\,\mathbf{X} = \mathbf{B}$$

$$\mathbf{A}_{ij} = \sum_m V_m \int_S \vec{e}_i \cdot \vec{\mathcal{E}}_m \, dS \int_S \vec{e}_j \cdot \vec{\mathcal{E}}_m^{\,*} \, dS + Z_s \delta_{ij}$$

$$\mathbf{B}_i = -\int_S \vec{e}_i \cdot \vec{E}_{inc} \, dS$$

coupling integrals (line integrals)

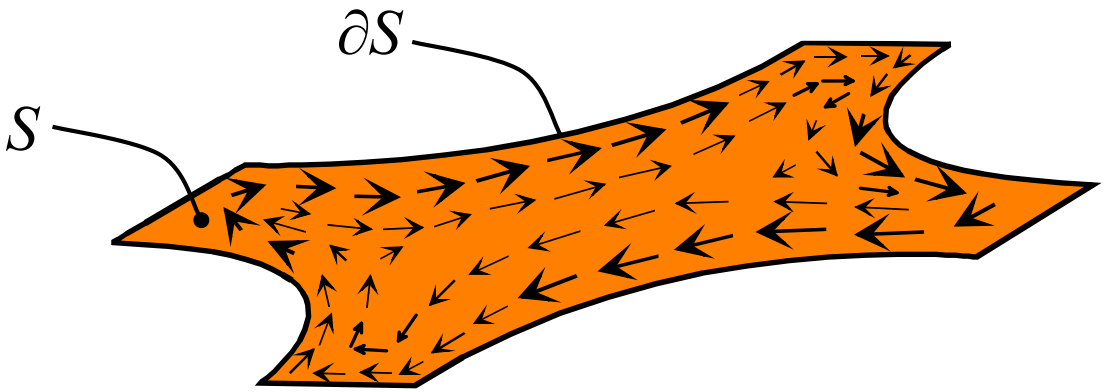
ROOF-TOPS



- apply to different shapes
- analytically available
- **large matrix problems**
- staircase approximation of curves and oblique segments

ENTIRE-DOMAIN BASIS FUNCTIONS

span the entire-domain S & satisfy the boundary condition



electric modal vectors of a waveguide with magnetic boundary

TM

$$\vec{e}_i = -\vec{u}_z \times \frac{\nabla_T \psi_i}{\kappa'_i}$$

$$\nabla_T^2 \psi_i + \kappa'^2_i \psi_i = 0 \qquad \text{in } S$$

$$\psi_i = 0 \qquad \text{on } \partial S$$

TE

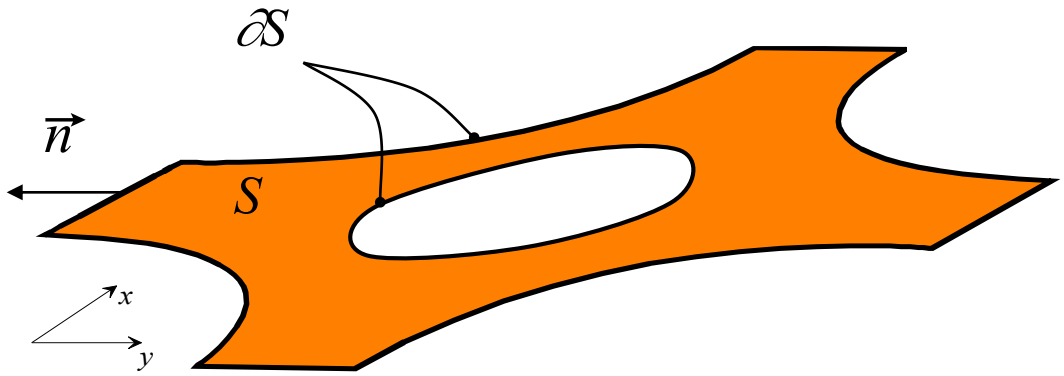
$$\vec{e}''_i = -\frac{\nabla_T \varphi_i}{\kappa''_i}$$

$$\nabla_T^2 \varphi_i + \kappa''^2_i \varphi_i = 0 \qquad \text{in } S$$

$$\frac{\partial \varphi_i}{\partial n} = 0 \qquad \text{on } \partial S$$

ENTIRE-DOMAIN BASIS FUNCTIONS

In the case of multiply connected patches, an additional set of basis functions is needed.



TEM

$$\vec{e}_i^{\,0} = -\vec{u}_z \times \nabla_T \psi_i$$

$$\nabla_T^2 \psi_i^0 = 0 \qquad \text{in } S$$

$$\psi_i^0 = \begin{cases} 1 & \text{on one inner boundary} \\ 0 & \text{elsewhere on the boundary} \end{cases}$$

THE BI-RME METHOD

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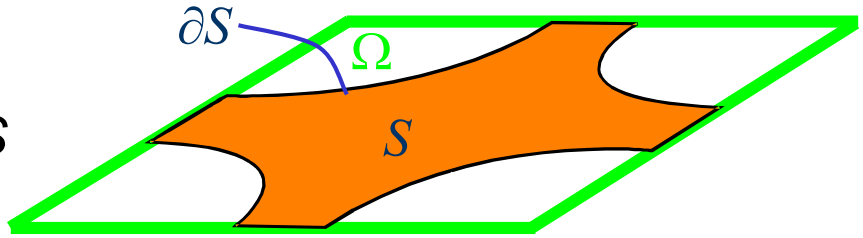
Basically the BI-RME method is a modified Boundary Element Method

The scalar potentials are considered in the enlarged domain Ω including *S* and are expressed as:



$$\psi(\vec{r}) = \int_{\partial S} G(\vec{r},\vec{s}\,') \left[\frac{\mathrm{d}\psi^-(\vec{s}\,')}{\mathrm{d}n} - \frac{\mathrm{d}\psi^+(\vec{s}\,')}{\mathrm{d}n} \right] d\ell' + \sum_i a_i \chi_i'(\vec{r})$$

Boundary Integral (BI) — **Resonant Mode Expansion (RME)**

$$\phi(\vec{r}) = \int_{\partial S} \left(\frac{\partial F_0(\vec{r},\vec{s}\,')}{\partial n'} + \kappa'' \frac{\partial F_1(\vec{r},\vec{s}\,')}{\partial n'} \right) \left[\phi^-(\vec{s}\,') - \phi^+(\vec{s}\,') \right] d\ell' + \kappa''^2 \sum_i b_i \chi_i''(\vec{r})$$

G, F_0, F_1 are static Green's functions of Ω;
χ', χ'' are TM / TE potentials of Ω

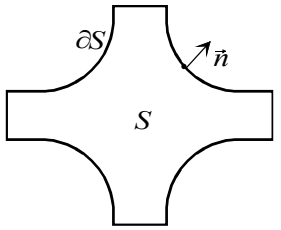
The Helmholtz equations for TM and TE modes are transformed into two linear integro-differential eigenvalue problems, whose solution gives *in one shot all the modes* in a prescribed frequency band.

COUPLING COEFFICIENTS

$$\int_S \vec{e}_i \cdot \vec{\mathcal{E}}_m^{\,*} \, \mathrm{d}S$$

SURFACE INTEGRALS IN S

LINE INTEGRALS ON ∂S



$$\int_S \vec{e}'_i \cdot \vec{\mathcal{E}}'^{\,*}_m \, \mathrm{d}S = 0$$

$$\int_S \vec{e}'_i \cdot \vec{\mathcal{E}}''^{\,*}_m \, \mathrm{d}S = \frac{k''_m}{\kappa'_i \left(\kappa'^{\,2}_i - k''^{\,2}_m\right)} \int_{\partial S} \frac{\partial \psi_i}{\partial n} \, \chi^*_m \, \mathrm{d}\ell$$

$$\int_S \vec{e}'_i \cdot \vec{\mathcal{E}}^{0\,*}_m \, \mathrm{d}S = 0$$

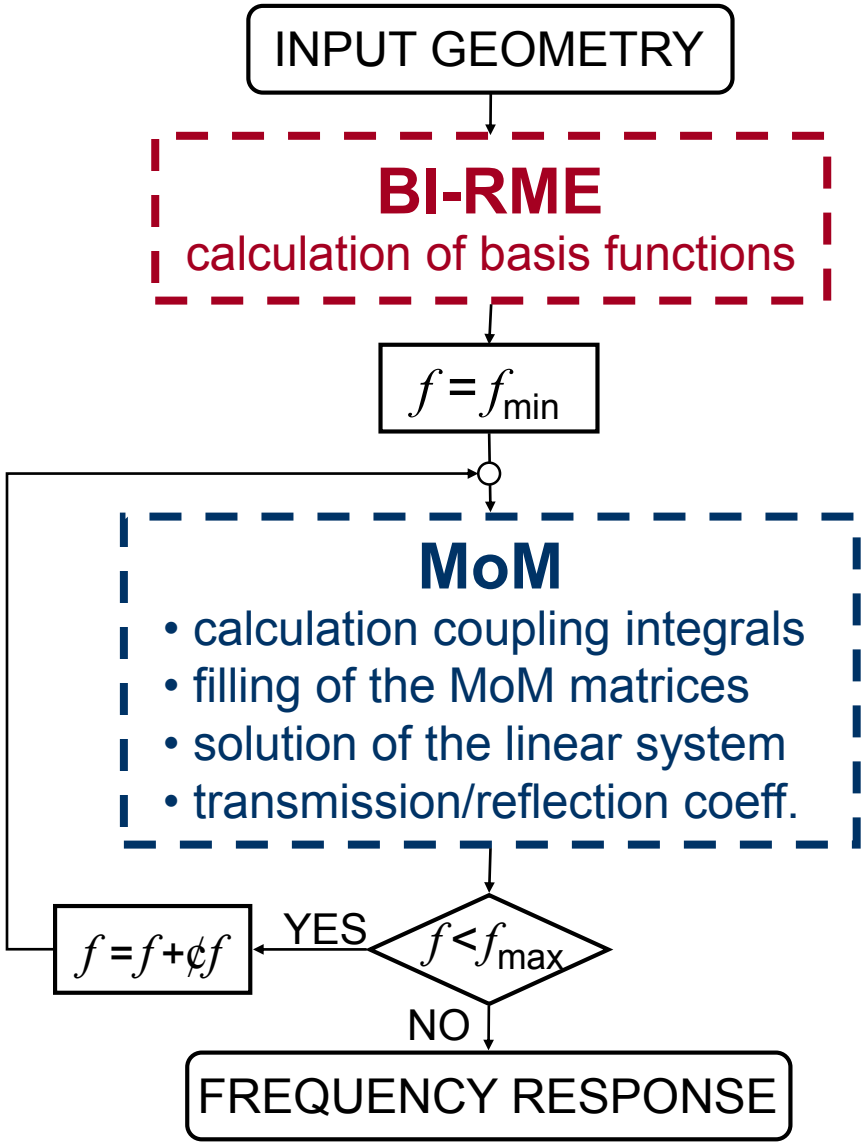
$$\int_S \vec{e}''_i \cdot \vec{\mathcal{E}}'^{\,*}_m \, \mathrm{d}S = \frac{\kappa''_i}{k'_m \left(\kappa''^{\,2}_i - k'^{\,2}_m\right)} \int_{\partial S} \varphi_i \, \frac{\partial \chi^*_{\mathrm{m}}}{\partial n} \, \mathrm{d}\ell$$

$$\int_S \vec{e}''_i \cdot \vec{\mathcal{E}}''^{\,*}_m \, \mathrm{d}S = \frac{1}{\kappa''_i \; k''_m} \int_{\partial S} \varphi_i \, \frac{\partial \chi^*_{\mathrm{m}}}{\partial \mathrm{t}} \, \mathrm{d}\ell$$

$$\int_S \vec{e}''_i \cdot \vec{\mathcal{E}}^{0}_m \, \mathrm{d}S = \frac{1}{\kappa''_i} \int_{\partial S} \varphi_i \, \frac{\partial \chi^0_m}{\partial n} \, \mathrm{d}\ell$$

M. Bozzi, G. Conciauro, and L. Perregrini,
"On the Evaluation of Modal Coupling Coefficients by Contour Integrals,"
IEEE Trans. on Microwave Theory & Techniques, July 2002.

STRUCTURE OF THE CODE



FSS WITH CROSS-SHAPED PATCHES

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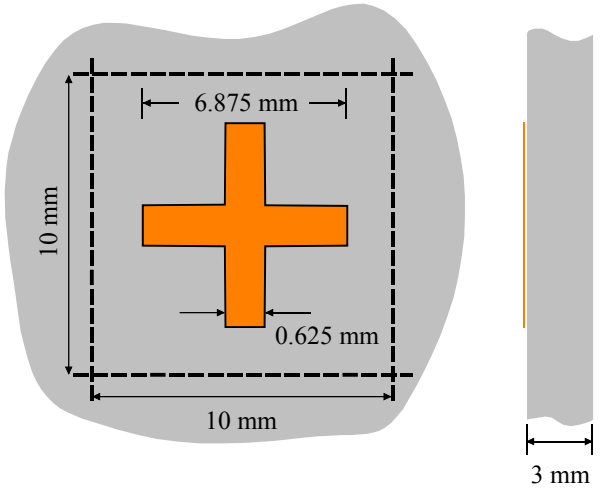
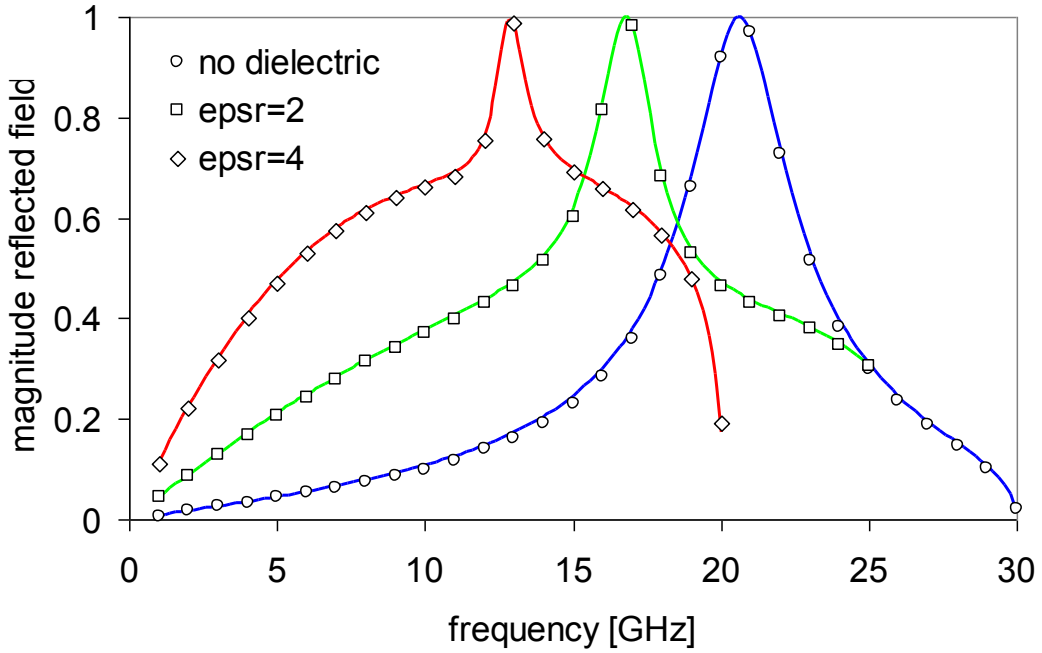
R. Mittra, C. Chan, and T. Cwik, *Proc. IEEE*, Dec. 1988.

A. S. Barlevy and Y. Rahmat–Samii, *Electromagnetics,* 1997.



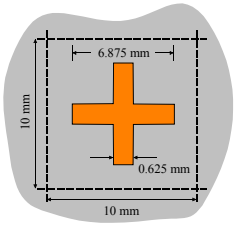
(normal incidence)
20 basis functions



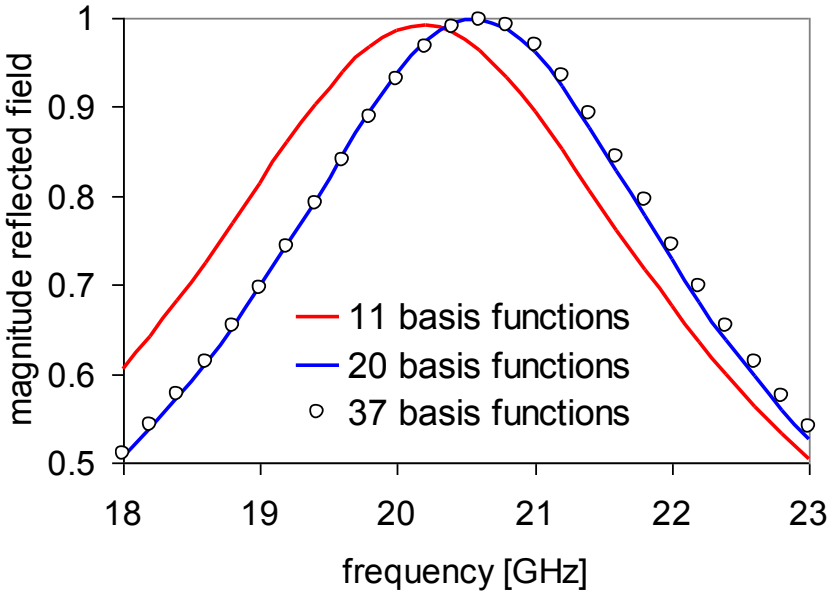
CPU time (Pentium 4 @ 1.7 *GHz*):

for computing the basis functions	6 sec
for the MoM problem	0.03 sec x 100 freq. points
overall time	9 sec

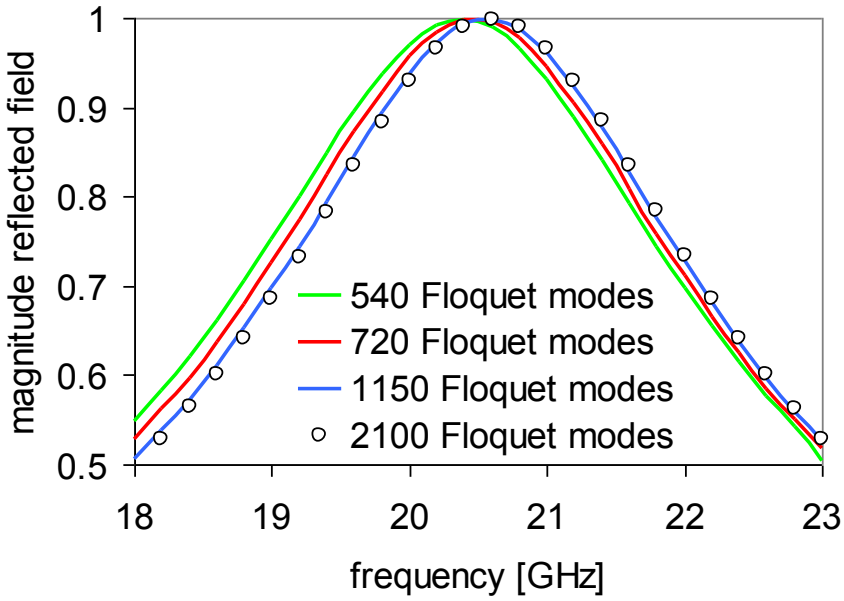
CONVERGENCE PROPERTIES



(no dielectric & normal incidence)



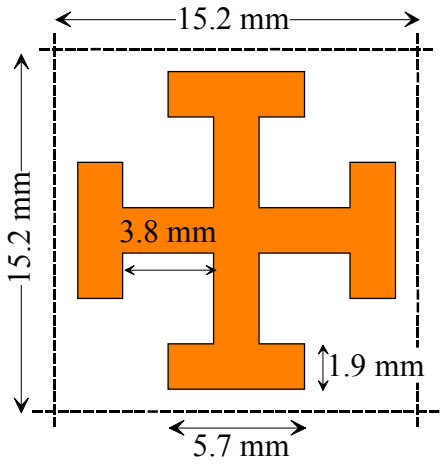
(with 1150 Floquet modes)



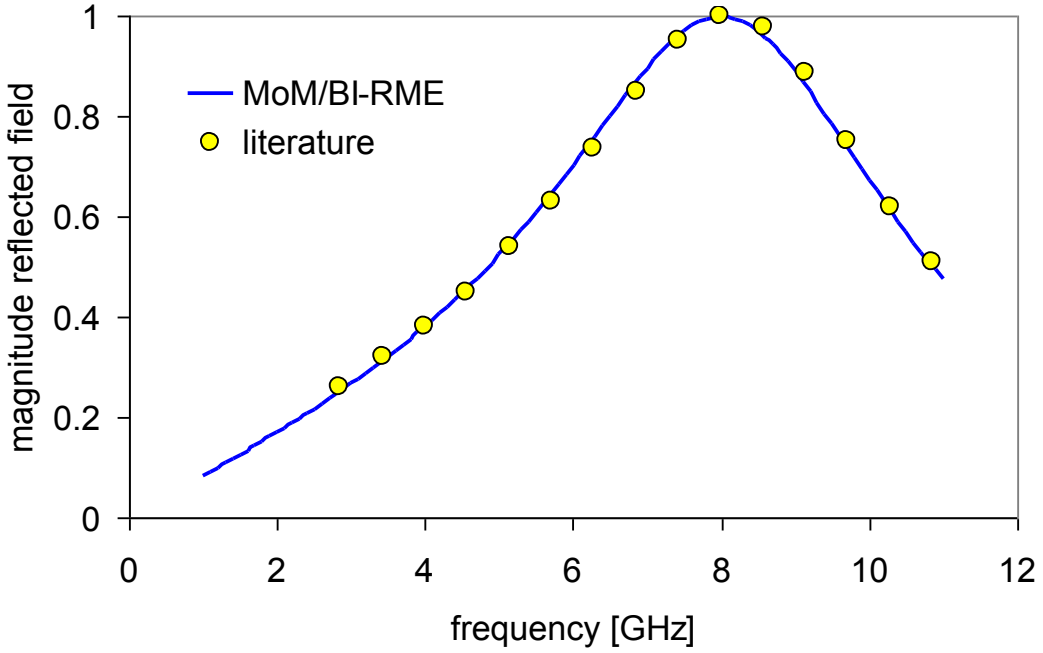
(with 20 basis functions)

FSS WITH JERUSALEM CROSSES

R. Mittra, C. Chan, and T. Cwik, *Proc. IEEE*, Dec. 1988.



(normal incidence)

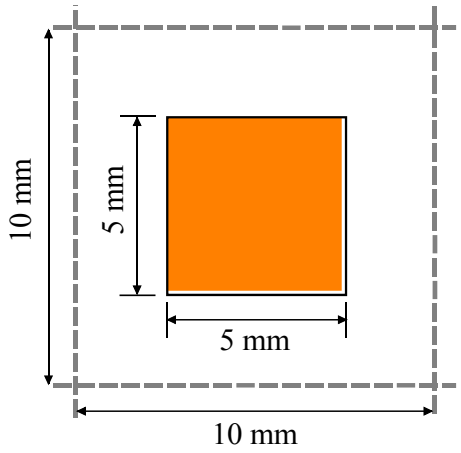


in the literature
584 roof-tops

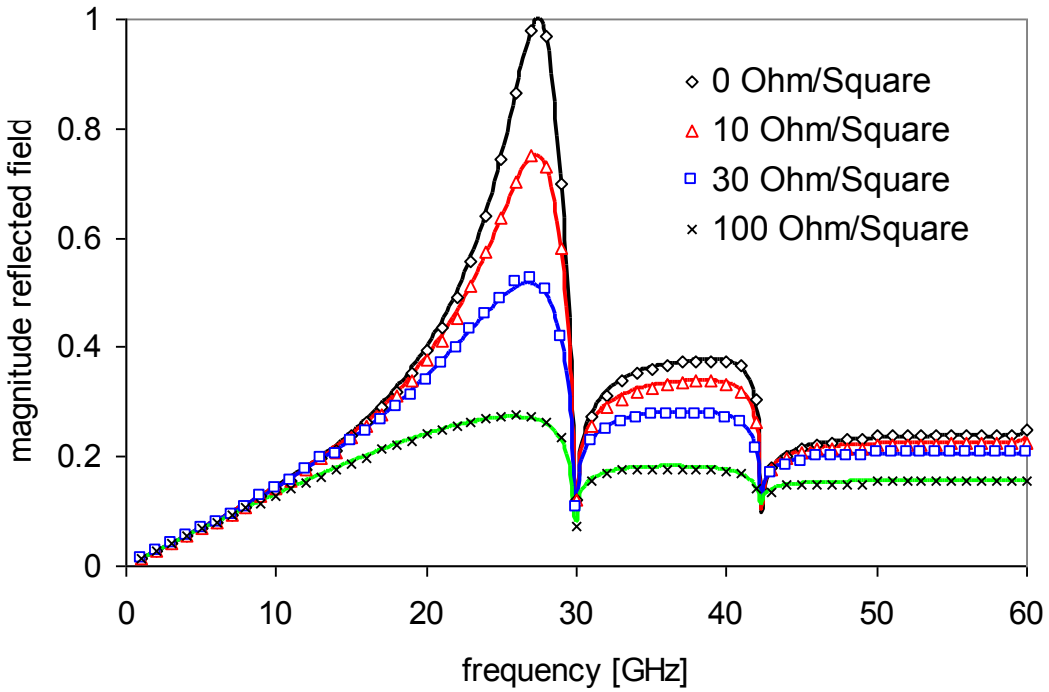
Entire-Domain Basis Functions:	54
Number of Floquet modes:	650
CPU time (Pentium III @ 1 *GHz*):	33 *sec* (100 frequency points)

PATCHES WITH METAL LOSSES

A. S. Barlevy and Y. Rahmat–Samii, *Electromagnetics,* 1997.



(normal incidence)



Entire-Domain Basis Functions:	28
Number of Floquet modes:	630
CPU time (Pentium III @ 1 *GHz*):	7 *sec* (296 frequency points)

DOUBLE-LAYER I-POLE FSS

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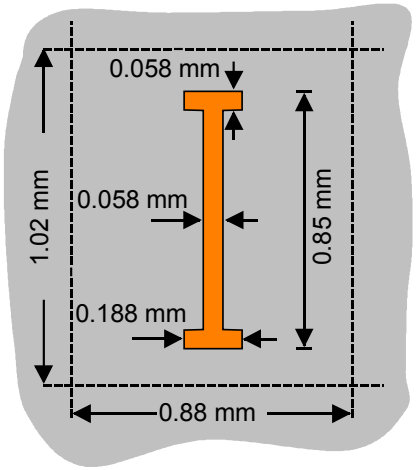
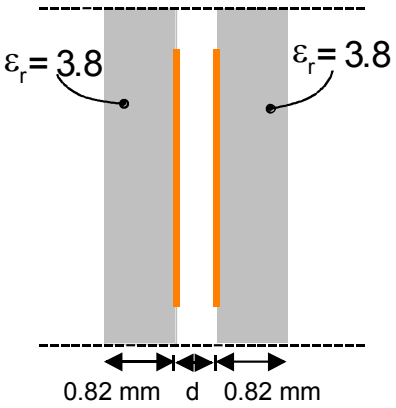
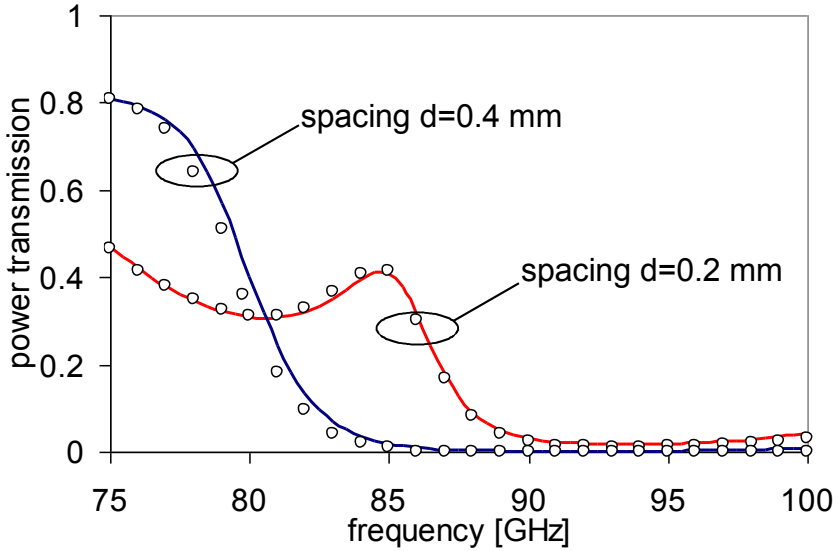
T. R. Schimert et al., *IEEE Trans. Microwave Theory & Techniques*, Feb. 1991.







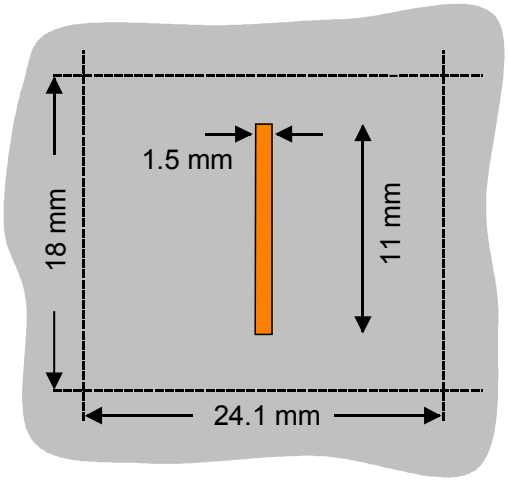
metal losses: $Z_S = 0.331 + j\,0.0888\ \Omega/\square$

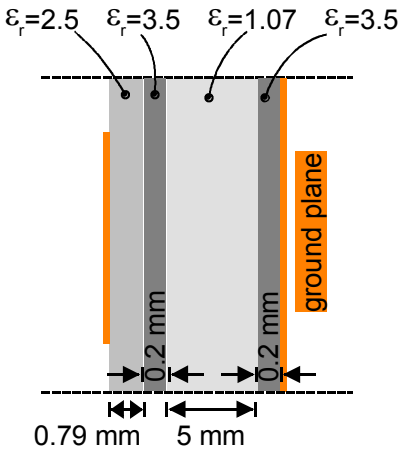
normal incidence, vertical polarization

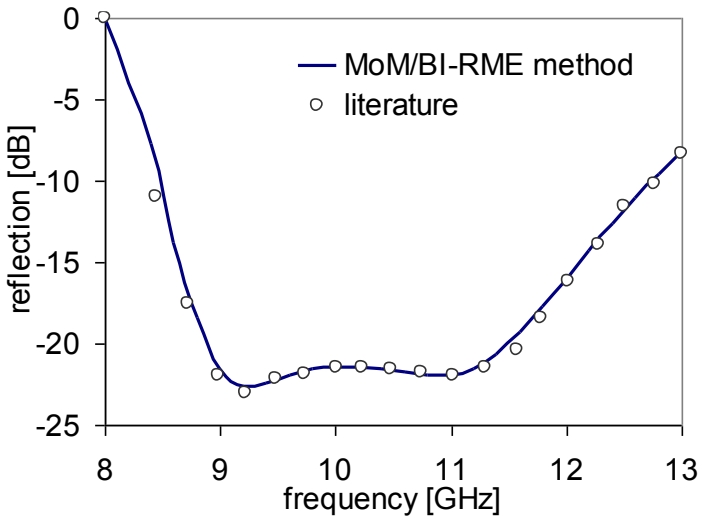
Entire-Domain Basis Functions:	19+19
Number of Floquet modes:	570
CPU time (Pentium 4 @ 1.7 *GHz*):	9 *sec* (100 frequency points)

REFLECTION GRATING

C. Wan and J. A. Encinar, *IEEE Trans. Antennas & Propag*, Nov. 1995.







incidence θ=30°, vertical polarization

Entire-Domain Basis Functions:	17
Number of Floquet modes:	880
CPU time (Pentium 4 @ 1.7 *GHz*):	15 *sec* (100 frequency points)

OUTLINE

- Introduction to frequency selective surfaces (FSS)
- Modeling of infinitely thin capacitive FSS
- Modeling of capacitive FSS with finite thickness
- Modeling of inductive FSS
- Conclusion

MOTIVATION OF THIS WORK

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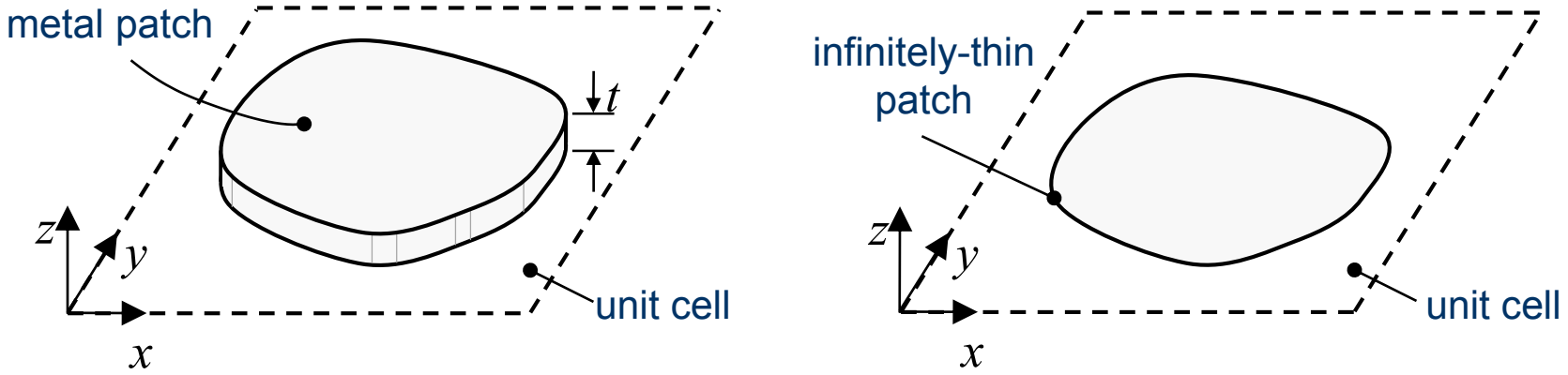
Printed periodic structures are usually modeled under the approximation of infinitely-thin metal patches.



typical thickness with PCB technology: t = 15μm ÷ 100μm

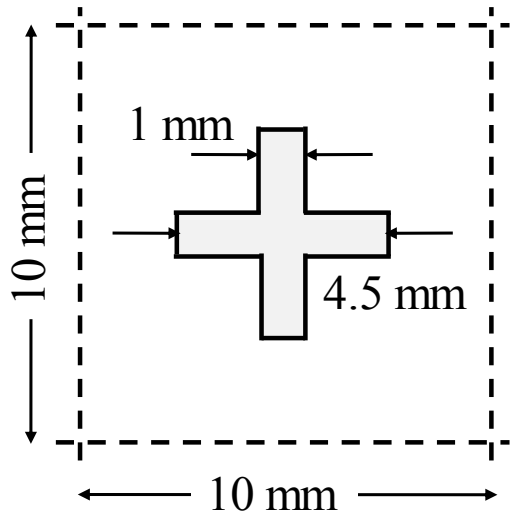
In the mm- and submm-wave frequency band:

- shift of the frequency response
- poor estimate of conductor losses

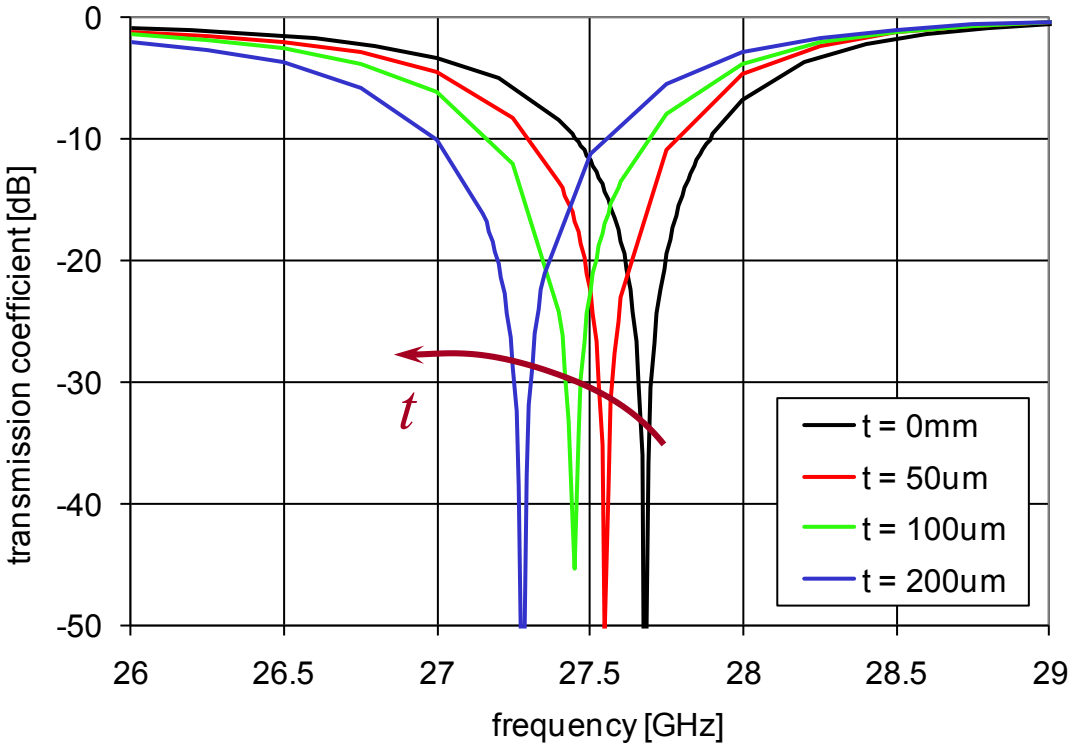
MOTIVATION OF THIS WORK

University of Pavia

Example of the effect of finite thickness t on the resonance frequency



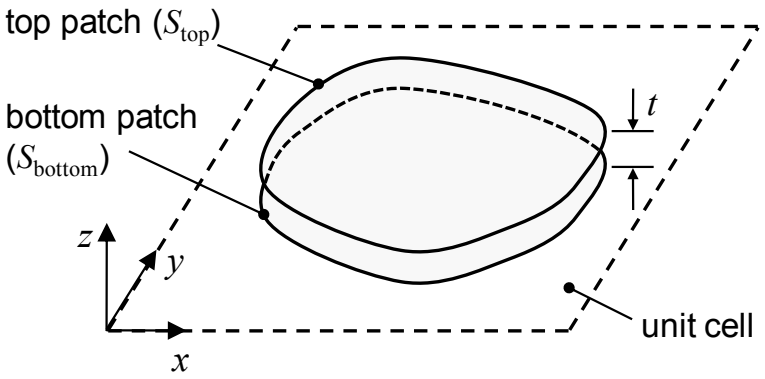
(analysis by Ansoft HFSS)



free-standing metal patch, normal incidence

THICK METAL PATCHES / 1

Two possible approaches have been investigated:



MODEL 1: **TWO THIN PATCHES**

- current $\vec{J}_{top}$ on the top patch
- current $\vec{J}_{bottom}$ on the bottom patch

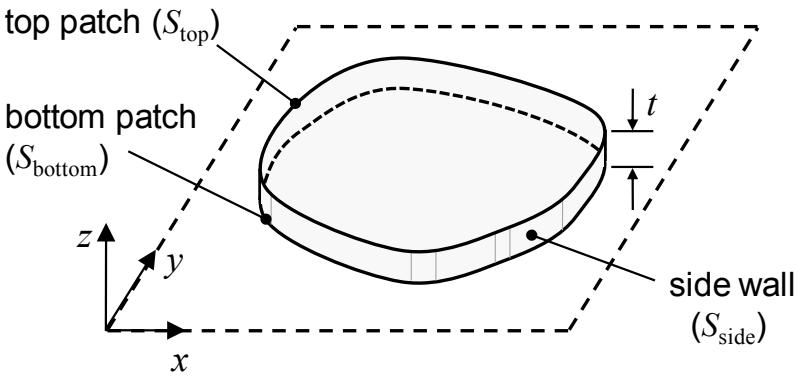
Problem: no lateral currents

MODEL 2: **THICK PATCH**

- inclusion of the side current $\vec{J}_{side}$

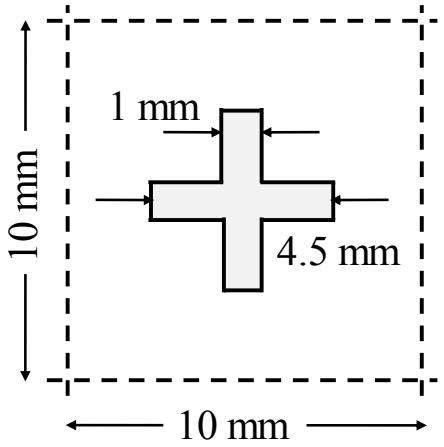
Problems: new basis functions?

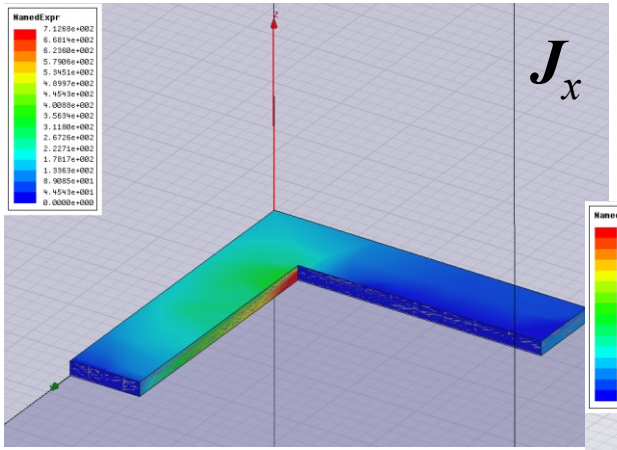
new components of the Green's Function?

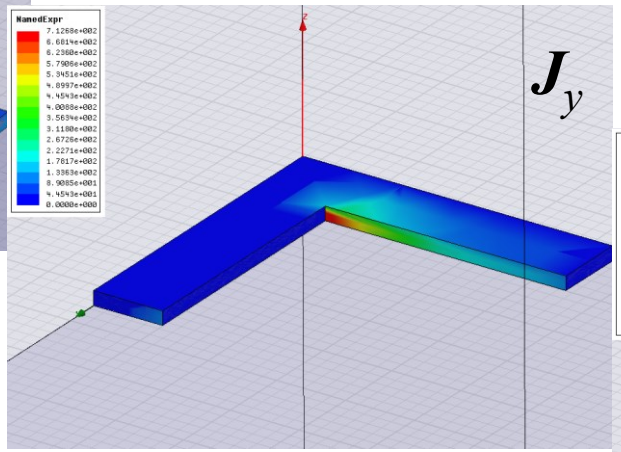


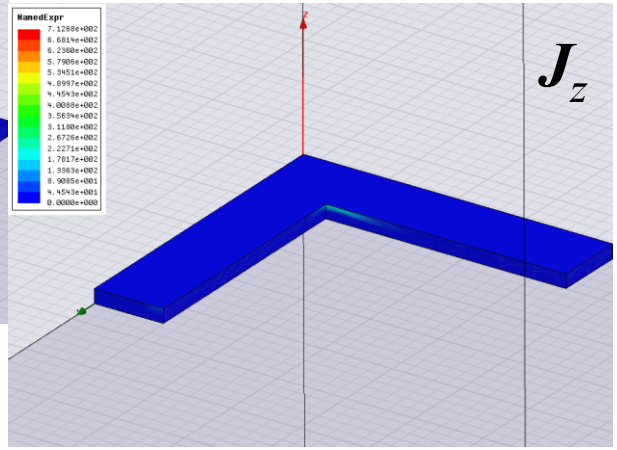
THICK METAL PATCHES / 2

Investigation of the current density distribution (Ansoft HFSS simulations, exploiting symmetries)









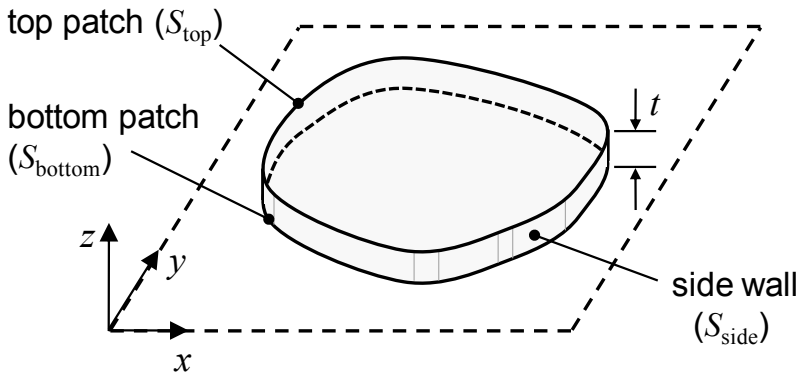
t = 100 μm

free-standing metal patch, normal incidence

The axial component of the current J_z is negligible!

THICK METAL PATCHES / 3

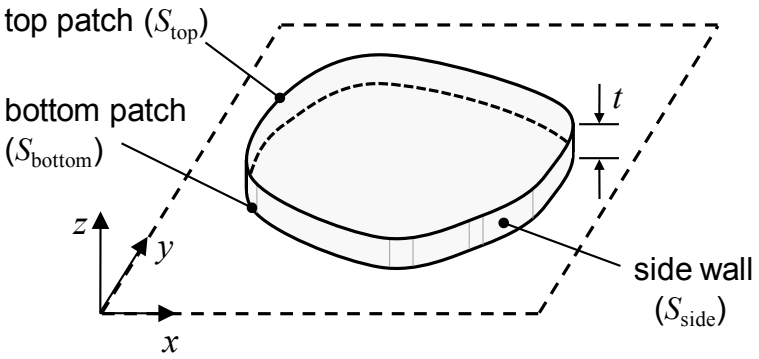
Since the axial current is negligible, no additional components of the Green's functions are required.



$$\underline{\boldsymbol{G}}(x,y,z,x',y',z') =$$
$$G_{xx}\boldsymbol{xx}'+G_{xy}\boldsymbol{xy}'+\cancel{G_{xz}\boldsymbol{xz}'}+G_{yx}\boldsymbol{yx}'+G_{yy}\boldsymbol{yy}'+\cancel{G_{yz}\boldsymbol{yz}'}+\cancel{G_{zx}\boldsymbol{zx}'}+\cancel{G_{zy}\boldsymbol{zy}'}+\cancel{G_{zz}\boldsymbol{zz}'}$$

INTEGRAL EQUATION

The integral equation is formulated by imposing the Leontovich condition on the surface $S = S_{\text{bottom}} + S_{\text{top}} + S_{\text{side}}$



$$\vec{\mathrm{E}}^{\text{inc}}(x,y,z) + \vec{\mathrm{E}}^{\text{scat}}(x,y,z) = Z_S\, \vec{J}(x,y,z)$$

$\vec{E}^{inc}$: incident field (fundamental Floquet modes)

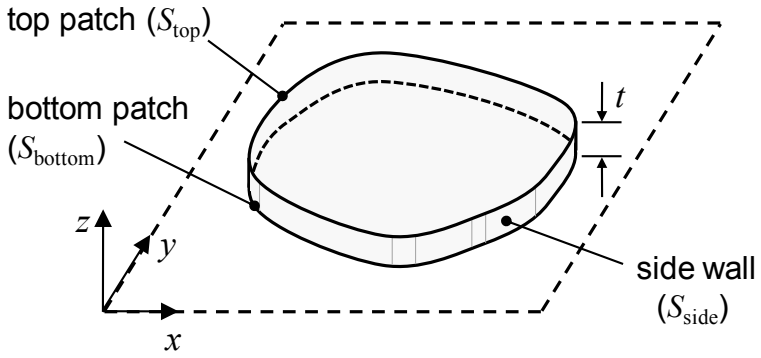
Z_s : sheet impedance (skin effect)

$\vec{\mathrm{E}}^{\text{scat}}(x,y,z) = \sum_{p=1}^{2} \int_{S_p} \vec{\vec{G}}(x,y,z;x',y',z') \cdot \vec{J}_p(x',y',z') dS'_p$: scattered field

where $S_1 = S_{\text{bottom}} + S_{\text{side}}$ and $S_2 = S_{\text{top}} + S_{\text{side}}$

BASIS FUNCTIONS

The original basis functions defined on S_{bottom} and S_{top} are extended to $S_1 = S_{\text{bottom}} + S_{\text{side}}$ and $S_2 = S_{\text{top}} + S_{\text{side}}$



$$\vec{J}_p(x',y',z') = \sum_{j=1}^{N} a_{p,j}\vec{e}_{p,j}(x,'y')f_p(z')$$

$\vec{e}_{p,j}$: waveguide modes (TE & TM waveguide modes)

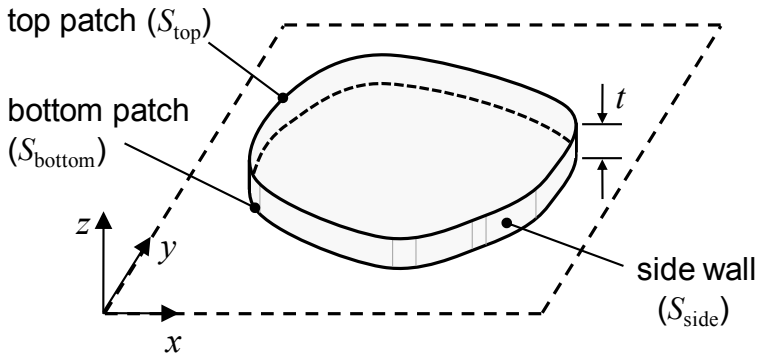
f_p : **scalar variation along** z

$$f_1(z') = \frac{z'-z_2}{z_1-z_2} \qquad f_2(z') = \frac{z'-z_1}{z_2-z_1} \qquad f_n(z') = \begin{cases} 1 & \text{if } z' = z_n \\ 0 & \text{if } z' = z_p \end{cases}$$

$a_{p,j}$: unknown coefficients

GREEN'S FUNCTION

The Green's function used in the integral equation is approximated, if the metal thickness t<<λ



$$\vec{\vec{G}}(x,y,z;x',y',z') = \sum_{m=1}^{M} V_m(z;z')\vec{E}_m(x,y)\vec{E}_m^*(x',y')$$

$\vec{E}_m$: Floquet modes

V_m: equivalent transmission line model

If t<<λ, the impedance does not vary significantly along z

$$\vec{\vec{G}}(x,y,z;x',y',z') \cong \sum_{m} V_m(z_n;z_p)\vec{E}_m(x,y)\vec{E}_m^*(x',y')$$

Final Matrix Problem

The final matrix problem, obtained from the application of the MoM, is:

$$\begin{bmatrix} \mathrm{A}^{1,1} & \mathrm{A}^{1,2} \\ \mathrm{A}^{2,1} & \mathrm{A}^{2,2} \end{bmatrix} \begin{bmatrix} \mathrm{X}^{\mathrm{I},1} & \mathrm{X}^{\mathrm{II},1} \\ \mathrm{X}^{\mathrm{I},2} & \mathrm{X}^{\mathrm{II},2} \end{bmatrix} = \begin{bmatrix} \mathrm{B}^{\mathrm{I},1} & \mathrm{B}^{\mathrm{II},1} \\ \mathrm{B}^{\mathrm{I},2} & \mathrm{B}^{\mathrm{II},2} \end{bmatrix}$$

$$\mathrm{A}_{i,j}^{n,p} = \sum_{m=1}^{M} V_m(z_n, z_p) \int_{S_n} \left[\vec{e}_{n,i}(x,y) f_n(z)\right] \cdot \vec{E}_m(x,y) dS_n \times \int_{S_p} \left[\vec{e}_{p,j}(x',y') f_p(z')\right] \cdot \vec{E}_m^*(x',y') dS_p' +$$
$$+ Z_S \delta_{np} \int_{S_n} \left[\vec{e}_{n,i}(x,y) f_n(z)\right] \cdot \left[\vec{e}_{p,j}(x,y) f_p(z)\right] dS_n$$

$$\mathrm{X}_j^p = a_{p,j}$$

$$\mathrm{B}_i^n = \int_{S_n} \left[\vec{e}_{n,i}(x,y) f_n(z)\right] \cdot \vec{\mathrm{E}}^{\mathrm{inc}}(x,y) dS_n$$

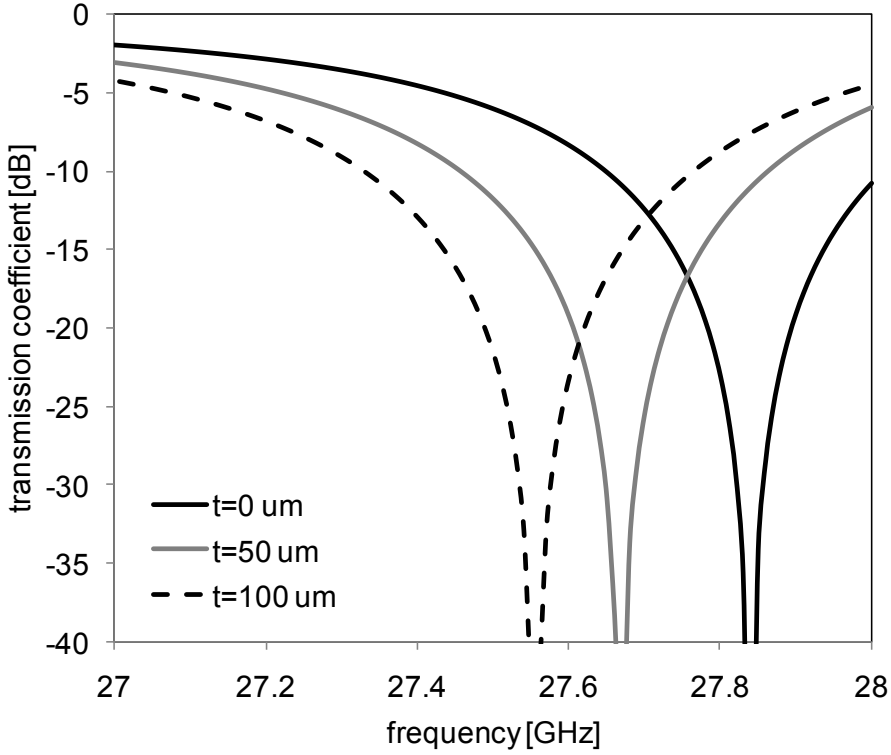
coupling coefficients (contour integrals)

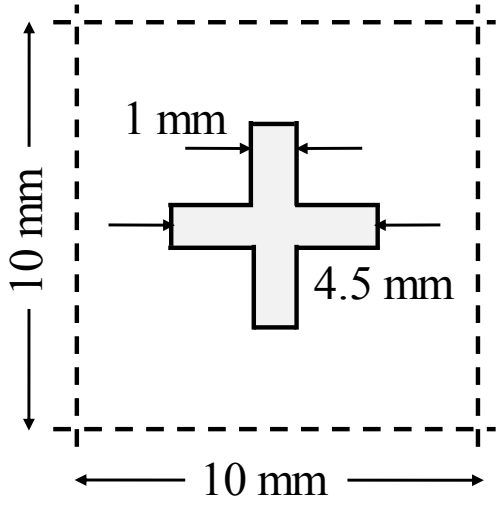
Compared to the thin-patch case, the number of unknowns is doubled.

VALIDATION EXAMPLE

University of Pavia

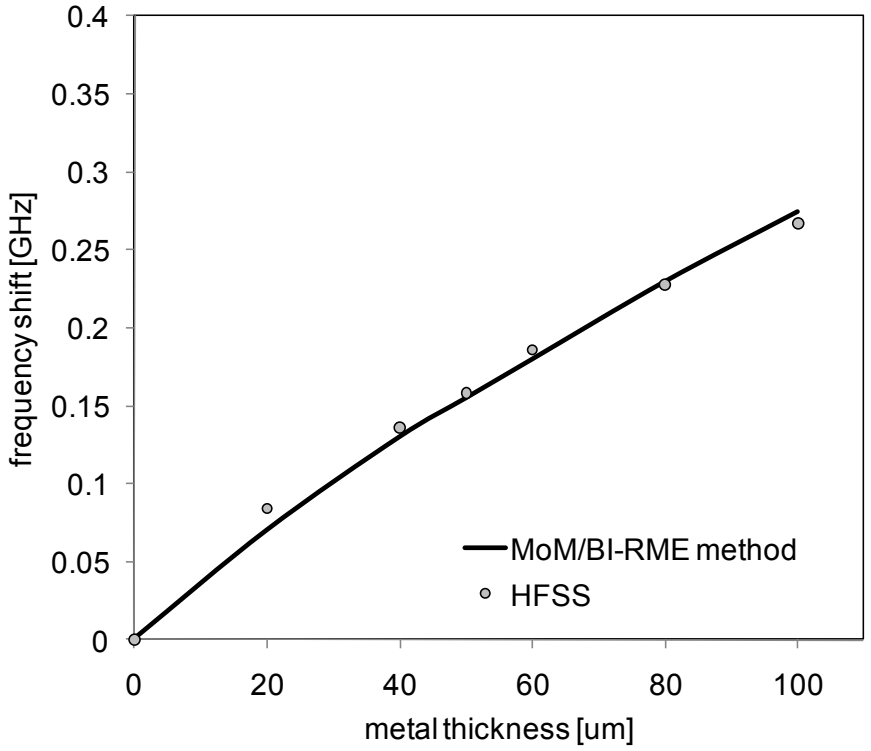
FSS with cross-shaped, free-standing metal patches (normal incidence)

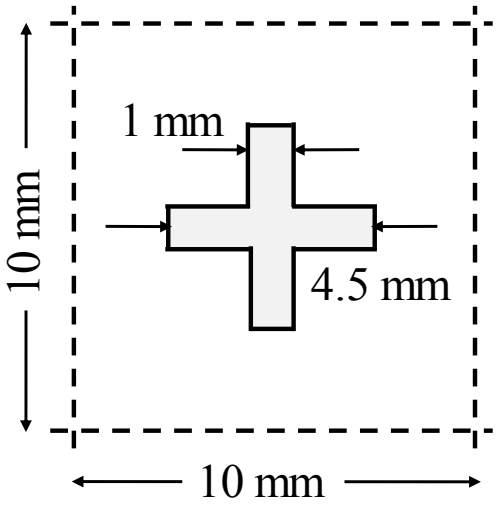




- 68 basis functions & 1100 Floquet modes
- overall CPU time (100 frequency points): 18 s
  (6 s basis functions, 0.12 x 100 s for the MoM problem)

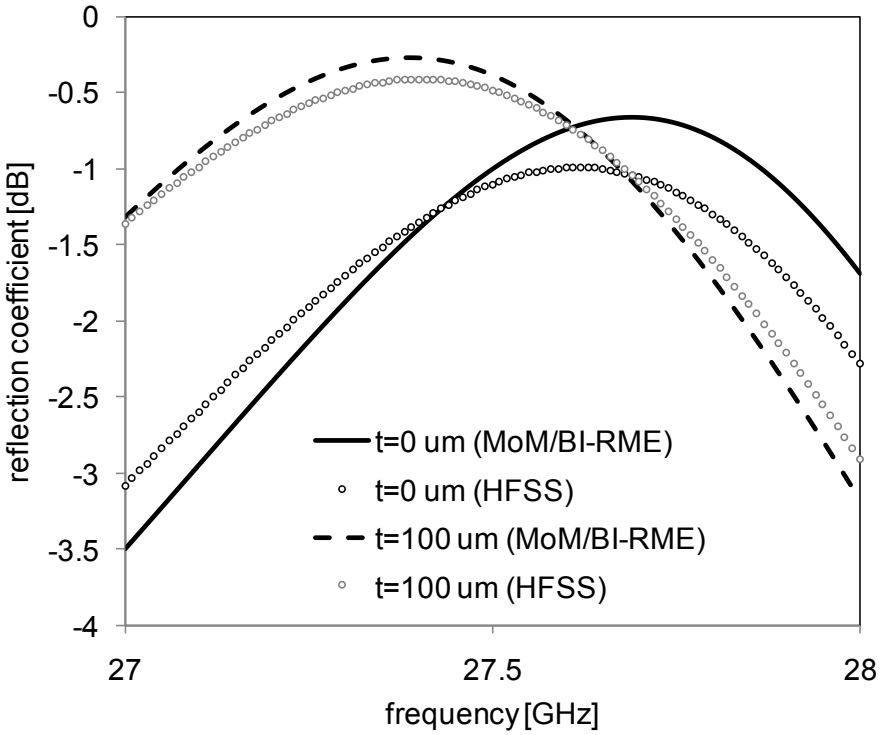
Frequency shift versus metal thickness

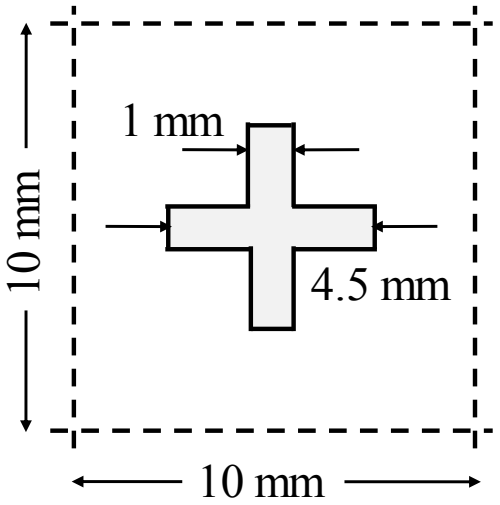




EFFECT ON LOSSES

Conductor losses are over-estimated by considering thin metal patches





$t = 100\ \mu m$

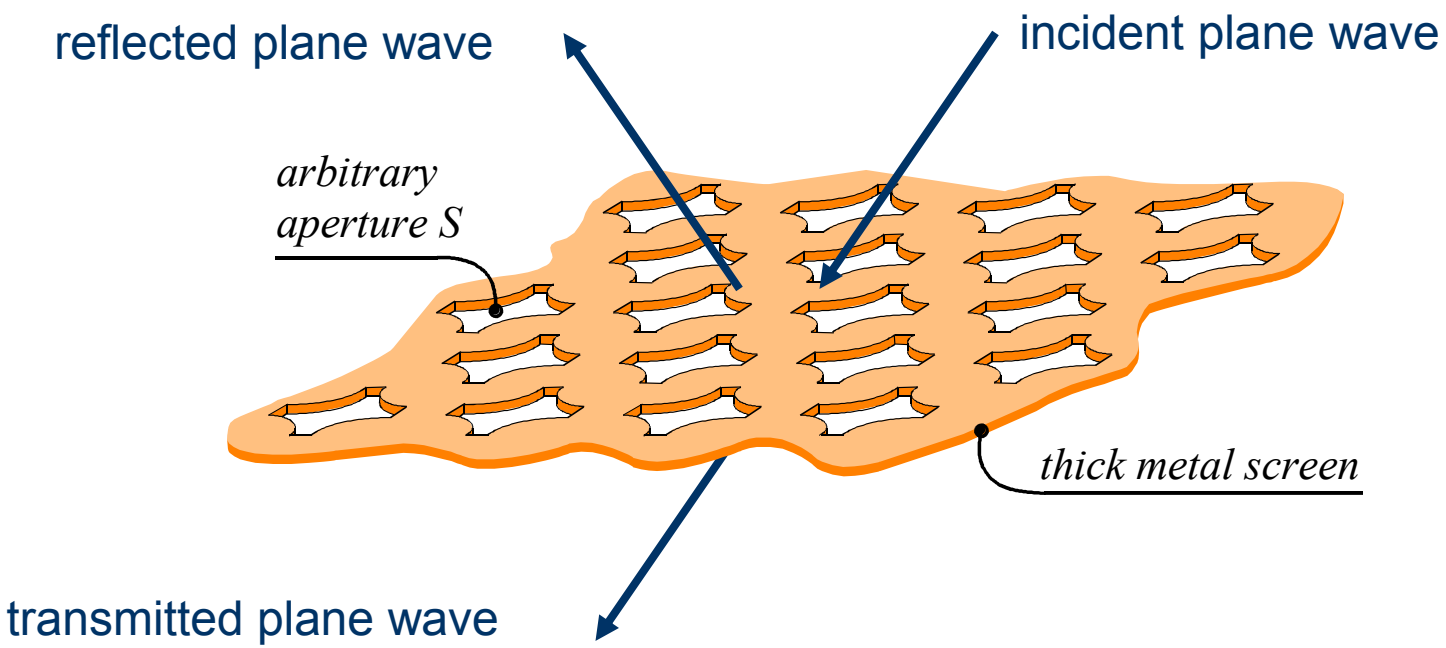
$\rho = 0.24\ 10^{-5}\ \Omega\ m$

OUTLINE

University of Pavia

- Introduction to frequency selective surfaces (FSS)
- Modeling of infinitely thin capacitive FSS
- Modeling of capacitive FSS with finite thickness
- Modeling of inductive FSS
- Conclusion

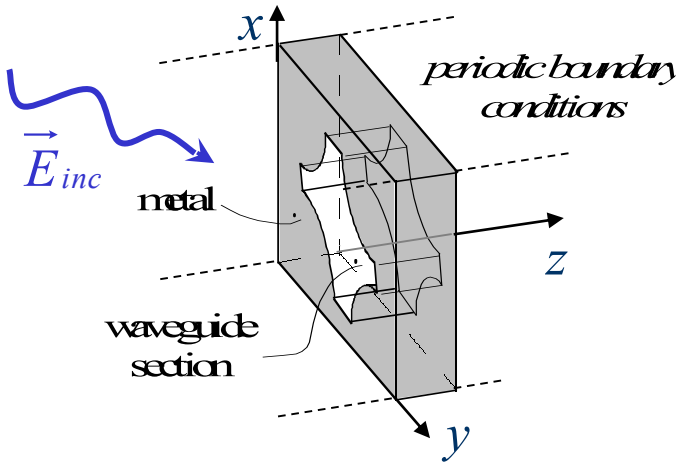
INDUCTIVE FSS

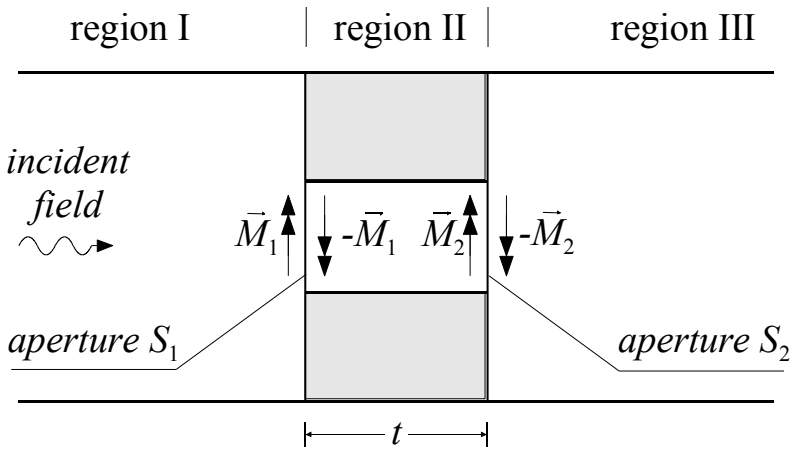


Thick metal screen perforated periodically with arbitrarily shaped apertures, illuminated by a uniform plane wave. No losses are considered in the metal.

THE MOM/BI-RME METHOD / 1

By using the equivalence theorem, the terminal wg sections are closed by perfect conductors, and magnetic current densities M_1 and M_2 are defined over them.





The fields in the three regions are expressed as Green's integrals, depending on M_1 and M_2. The Green's function is written in the form of a modal expansion.

region I and III ⇨ FLOQUET MODES

region II ⇨ WAVEGUIDE MODES

THE MOM/BI-RME METHOD / 2

Choice of the basis functions:

$$\vec{M}_1 = \sum_{q=1}^{Q} X_q \vec{h}_q \qquad \vec{M}_2 = \sum_{q=1}^{Q} Y_q \vec{h}_q$$

waveguide modal vectors

Final matrix problem obtained by using the MoM:

$$\begin{bmatrix} \mathbf{A}+\mathbf{B} & \mathbf{C} \\ \mathbf{C} & \mathbf{A}+\mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \end{bmatrix} = \begin{bmatrix} \mathbf{D} \\ \mathbf{0} \end{bmatrix}$$

$$\mathbf{A}_{rq} = \sum_m V_m \int_S \vec{h}_r \cdot \vec{\mathcal{H}}_m \, dS \qquad \int_S \vec{h}_q \cdot \vec{\mathcal{H}}_m^* \, dS$$

$$\mathbf{B}_{rq} = \begin{cases} -j\, y_q / \tan(\gamma_q t) & if\ q = r \\ 0 & otherwise \end{cases}$$

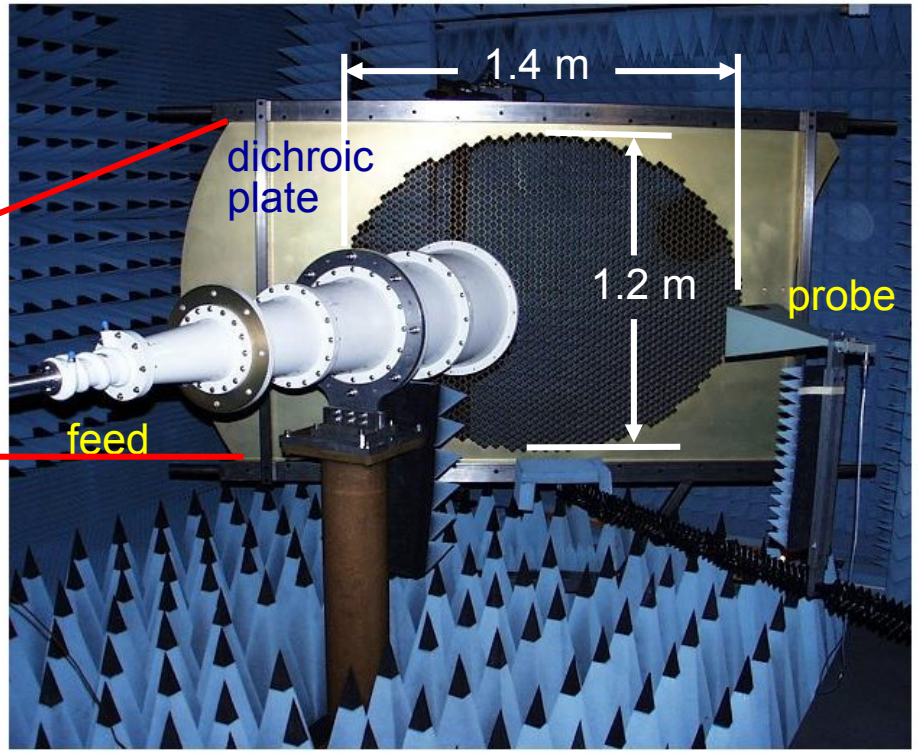
$$\mathbf{C}_{rq} = \begin{cases} j\, y_q / \sin(\gamma_q t) & if\ q = r \\ 0 & otherwise \end{cases}$$

$$\mathbf{D}_r = 2 \int_S \vec{h}_r \cdot \vec{H}_{inc} \, dS$$

coupling coefficients

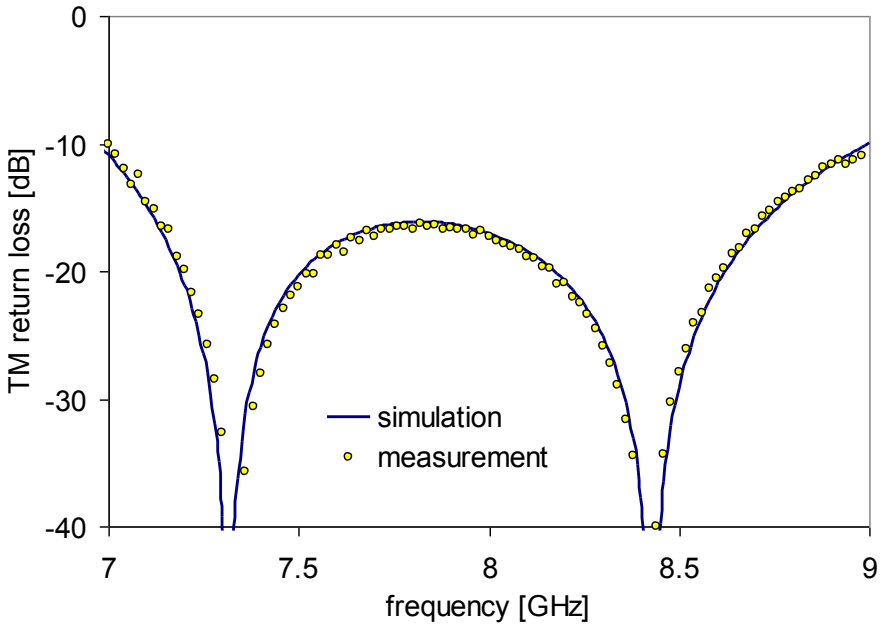
S/X-Band Dichroic Mirror

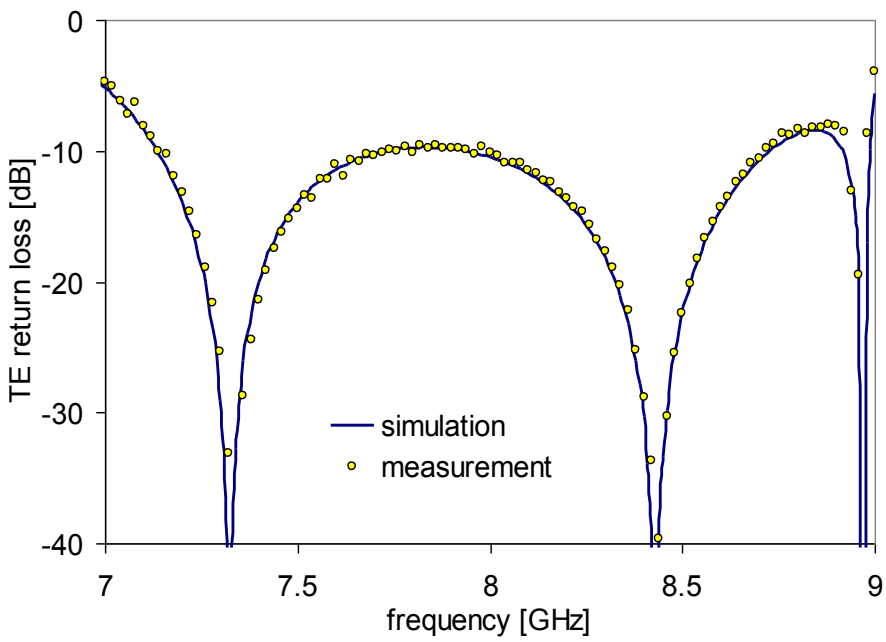
This FSS reflects at 2.025-2.3 GHz (S-band) and is transparent at 7.145-7.235 GHz & 8.4-8.5 GHz (X-band).
It operates in dual circular polarization.



DEEP SPACE ANTENNA OF ESA IN PERTH, AUSTRALIA

S/X-Band Dichroic Mirror

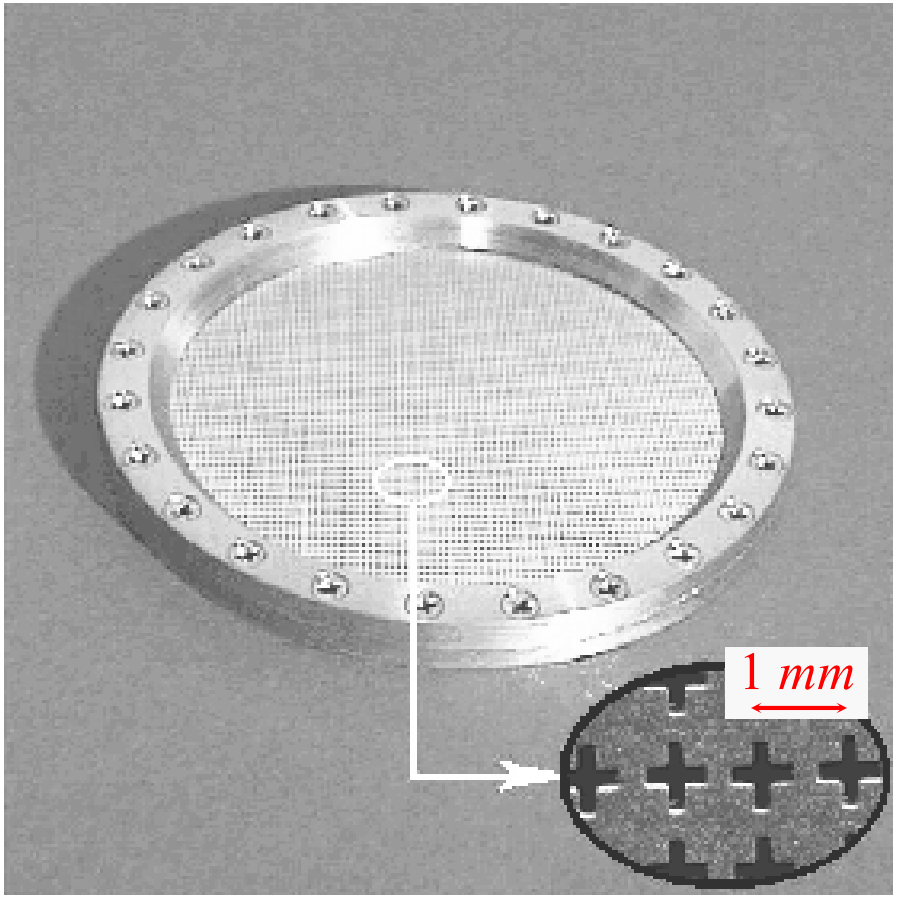




Entire-Domain Basis Functions:	34
Number of Floquet modes:	400
CPU time (Pentium 4 @ 1.7 *GHz*):	7 *sec* (100 frequency points)

P. Besso, M. Bozzi, L. Perregrini, L. Salghetti Drioli, and W. Nickerson, "Deep Space Antenna for Rosetta Mission: Design and Testing of the S/X-Band Dichroic Mirror," *IEEE Transactions on Antennas and Propagation*, Vol. AP-51, No. 3, pp. 388-394, March 2003..

BAND-PASS FILTER @ 280 GHZ

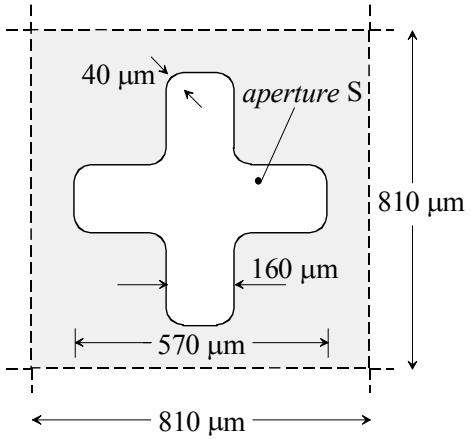


- The filter consists of a 10 μm free–standing copper foil
- It was fabricated by galvanizing growth (*University of Erlangen–Nürnberg, Germany*)

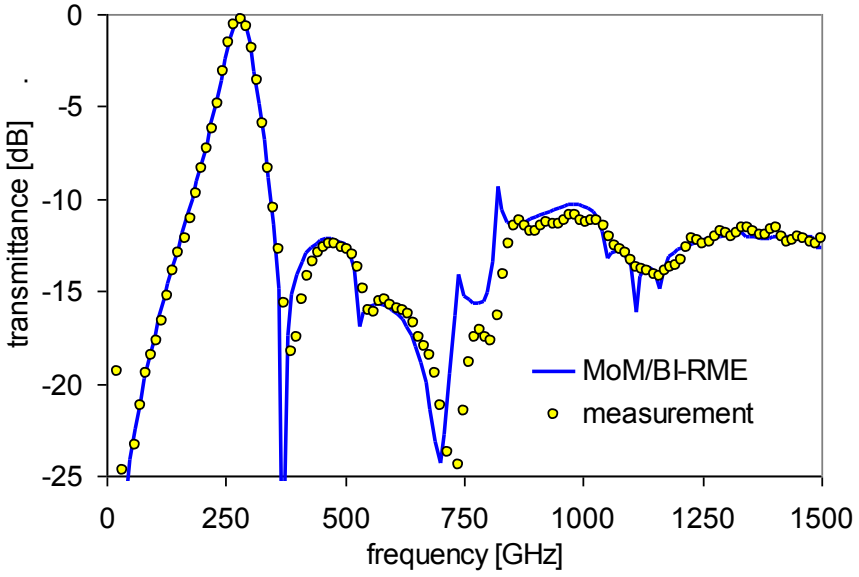
(normal incidence)

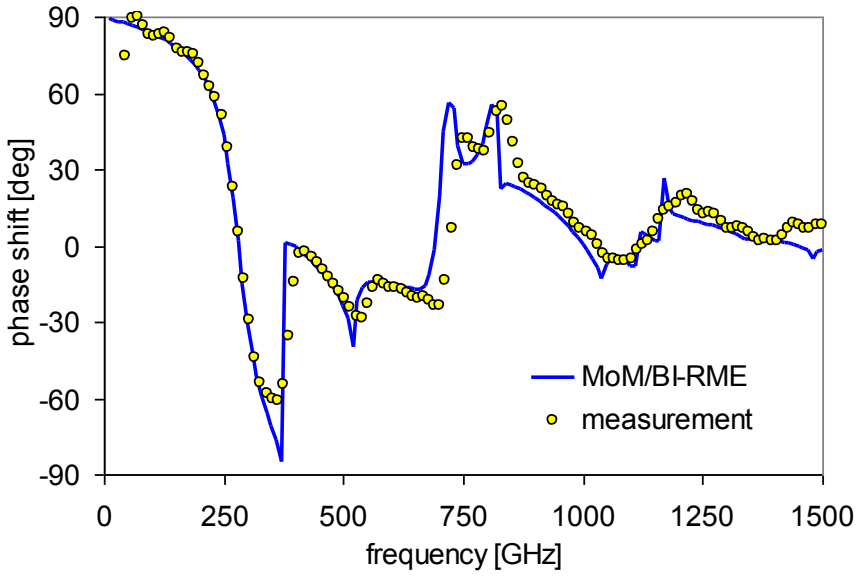
BAND-PASS FILTER @ 280 GHZ

University of Pavia



Entire-Domain Basis Functions: 30
Floquet modes: 420
CPU time (160 frequency points): 6 *sec*



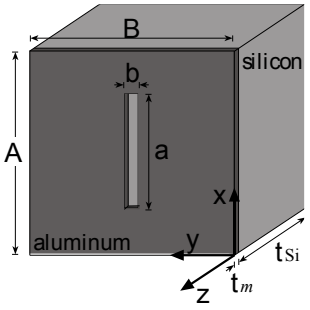


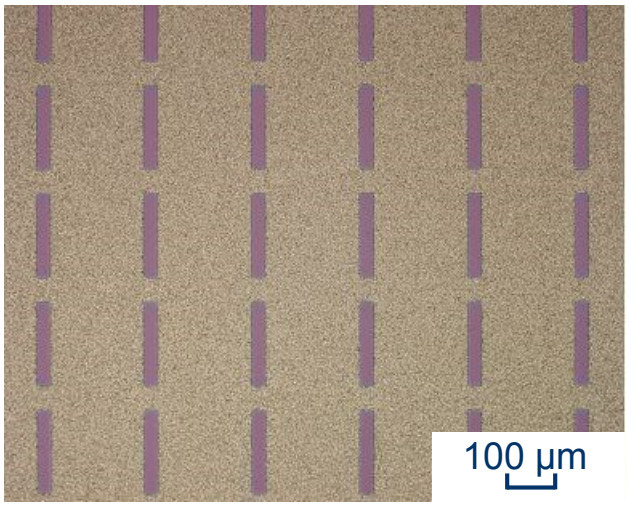
M. Bozzi, L. Perregrini, J. Weinzierl, and C. Winnewisser, "Efficient Analysis of Quasi-Optical Filters by a Hybrid MoM/ BI-RME Method," *IEEE Transactions on Antennas and Propagation*, Vol. AP–49, No. 7, pp. 1054–1064, July 2001.

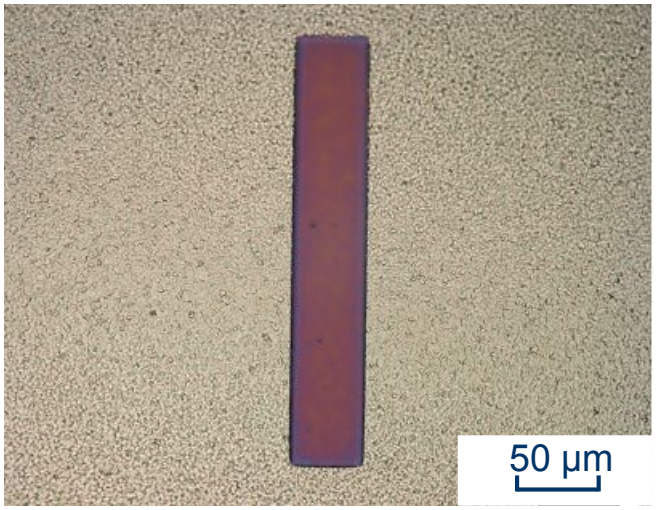
FSS ON SI SUBSTRATE @ 300 GHZ

Aluminum was structured on a silicon wafer.

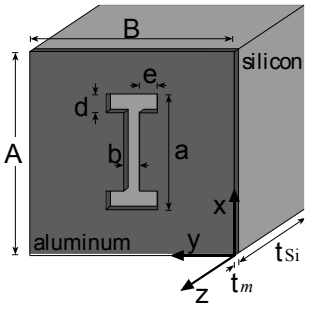
SLOT

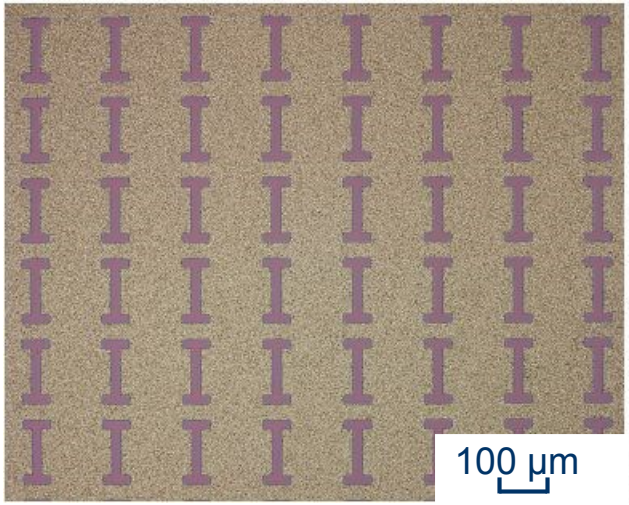


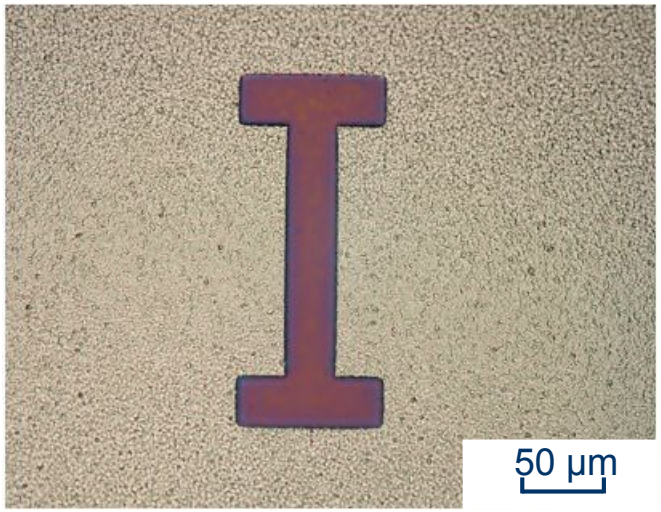




DOGBONE



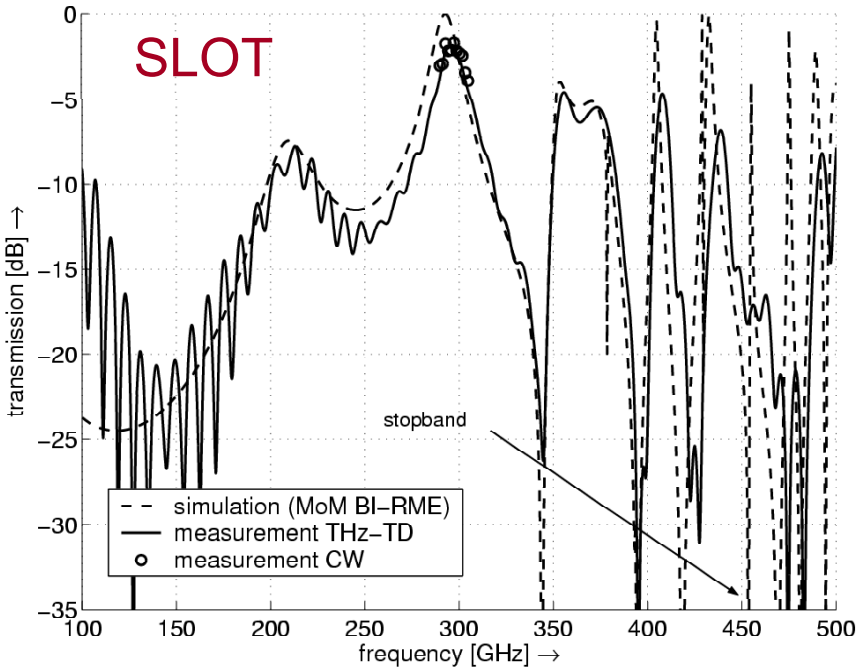


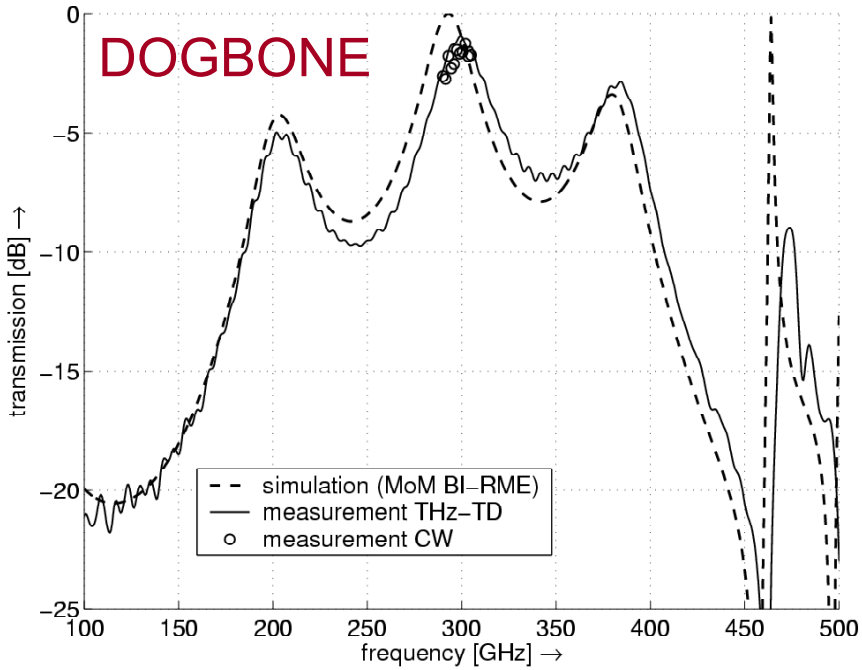


FSS ON SI SUBSTRATE @ 300 GHZ

Insertion loss 1.0 dB.

There is a systematic frequency shift of about 2%.





S. Biber, M. Bozzi, O. Guenther, L. Perregrini, and L.-P. Schmidt, "Design and Testing of Frequency Selective Surfaces on Silicon Substrates for Sub-mm Wave Applications," *IEEE Transactions on Antennas and Propagation*, Vol. AP-54, No. 9, pp. 2638-2645, Sept. 2006.

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