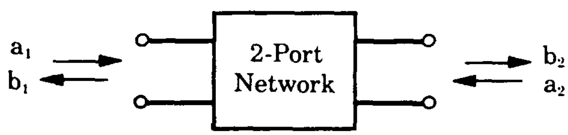
Chapter 2. Basic Microwave Review II

This second portion of Agilent Technologies' Basic Microwave Review will introduce some additional concepts that are used in high frequency amplifier design.

Scattering Transfer Parameters

Let's now proceed to a set of network parameters used when cascading networks. We recall that we developed the S-parameters by defining the reflected waves as dependent variables, and incident waves as independent variables (Fig. 39a). We now want to rearrange these equations such that the input waves a_1 and b_1 are the dependent variables and the output waves a_2 and b_2 the independent variables. We'll call this new parameter set scattering transfer parameters or T-parameters (Fig. 39b).

a)



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

b)

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$

Figure 39

The T-parameters can be developed by manipulating the S-parameter equations into the appropriate form. Notice that the denominator of each of these terms is S_{21} (Fig. 40).

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} -\dfrac{S_{11}S_{22} - S_{12}S_{21}}{S_{21}} & \dfrac{S_{11}}{S_{21}} \\ -\dfrac{S_{22}}{S_{21}} & \dfrac{1}{S_{21}} \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{T_{12}}{T_{22}} & \dfrac{T_{11}T_{22} - T_{12}T_{21}}{T_{22}} \\ \dfrac{1}{T_{22}} & -\dfrac{T_{21}}{T_{22}} \end{bmatrix}$$

Figure 40

We can also find the S-parameters as a function of the T-parameters.

While we defined the T-parameters in a particular way, we could have defined them such that the **output** waves are the dependent variables and the **input** waves are the independent variables. This alternate definition can result in some problems when designing with active unilateral devices (Fig. 41).

$$T_A] = \begin{bmatrix} \dfrac{S_{12}S_{21} - S_{11}S_{22}}{S_{12}} & \dfrac{S_{22}}{S_{12}} \\ -\dfrac{S_{11}}{S_{12}} & \dfrac{1}{S_{12}} \end{bmatrix}$$

Figure 41

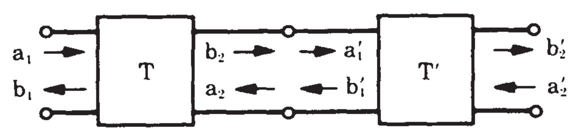
Using the alternate definition for the transfer parameters, the denominator in each of these terms is S_{12} rather than S_{21} as we saw earlier.

Working with amplifiers, we often assume the device to be unilateral, or $S_{12} = 0$. This would cause this alternate T-parameter set to go to infinity.

Both of these definitions for T-parameters can be encountered in practice. In general, we prefer to define the T-parameters with the output waves as the dependent variables, and the input waves as the independent variables.

We use this new set of transfer parameters when we want to cascade networks–two stages of an amplifier, or an amplifier with a matching network for example (Fig. 42a). From measured S-parameter data, we can define the T-parameters for the two networks. Since the output waves of the first network are identical to the input waves of the second network, we can now simply multiply the two T-parameter matrices and arrive at a set of equations for the overall network (Fig. 42b).

a)



b)

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \text{ and } \begin{bmatrix} b'_1 \\ a'_1 \end{bmatrix} = \begin{bmatrix} T'_{11} & T'_{12} \\ T'_{21} & T'_{22} \end{bmatrix} \begin{bmatrix} a'_2 \\ b'_2 \end{bmatrix}$$

$$\text{but } \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} b'_1 \\ a'_1 \end{bmatrix}$$

$$\text{Therefore } \begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} T'_{11} & T'_{12} \\ T'_{21} & T'_{22} \end{bmatrix} \begin{bmatrix} a'_2 \\ b'_2 \end{bmatrix}$$

Figure 42

Since matrix multiplication is, in general, not commutative, these T-parameter matrices must be multiplied in the proper order. When cascading networks, we'll have to multiply the matrices in the same order as the networks are connected. Using the alternate definition for T-parameters previously described, this matrix multiplication must be done in reverse order.

This transfer parameter set becomes very useful when using computer-aided design techniques where matrix multiplication is an easy task.

Signal Flow Graphs

If we design manually, however, we can use still another technique–signal flow graphs–to follow incident and reflected waves through the networks. This is a comparatively new technique for microwave network analysis.

A. Rules

We'll follow certain rules when we build up a network flow graph.

1. Each variable, a_1, a_2, b_1, and b_2 will be designated as a node.

2. Each of the S-parameters will be a branch.

3. Branches enter dependent variable nodes, and emanate from the independent variable nodes.

4. In our S-parameter equations, the reflected waves b_1 and b_2 are the dependent variables and the incident waves a_1 and a_2 are the independent variables.

5. Each node is equal to the sum of the branches entering it.

Let's now apply these rules to the two S-parameter equations (Fig. 43a). The first equation has three nodes: b_1, a_1, and a_2. b_1 is a dependent node and is connected to a_1 through the branch S_{11} and to node a_2 through the branch S_{12}. The second equation is constructed similarly. We can now overlay these to have a complete flow graph for a two-port network (Fig. 43b).

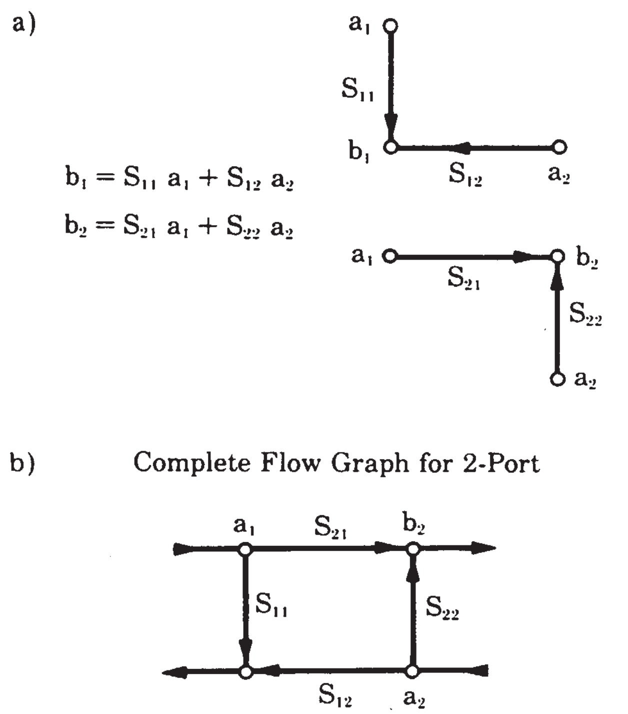


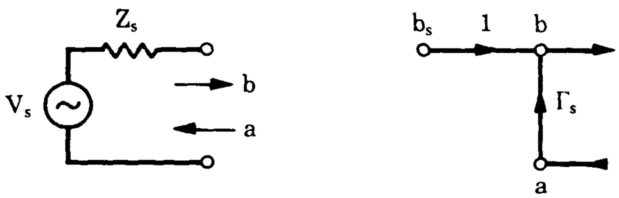
Figure 43

The relationship between the traveling waves is now easily seen. We have a_1 incident on the network. Part of it transmits through the network to become part of b_2. Part of it is reflected to become part of b_1. Meanwhile, the a_2 wave entering port two is transmitted through the network to become part of b_1 as well as being reflected from port two as part of b_2. By merely following the arrows, we can tell what's going on in the network.

This technique will be all the more useful as we cascade networks or add feedback paths.

B. Application of Flow Graphs

Let's now look at several typical networks we will encounter in amplifier designs. A **generator** with some internal voltage source and an internal impedance will have a wave emanating from it. The flow graph for the generator introduces a new term, b_S (Fig. 44). It's defined by the impedance of the generator. The units in this equation look peculiar, but we have to remember that we originally normalized the traveling waves to $\sqrt{Z_o}$. The magnitude of b_S squared then has the dimension of power.



$$b_s = \frac{V_s \sqrt{Z_o}}{Z_s + Z_o}$$

Figure 44

For a **load,** the flow graph is simply Γ_L, the complex reflection coefficient of the load (Fig. 45).

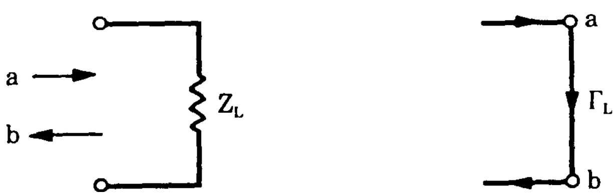


Figure 45

When the load is connected to the generator, we see a wave emanating from the generator incident on the load, and a wave reflected back to the generator from the load (Fig. 46).

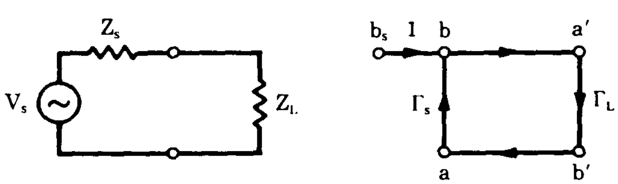


Figure 46

To demonstrate the utility of flow graphs, let's embed a two-port network between a source and a load. Combining the examples we have seen, we can now draw a flow graph for the system (Fig. 47).

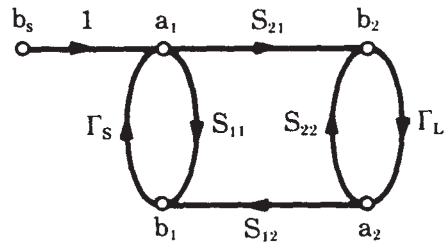


Figure 47

We can now apply the rule known as Mason's rule (or as it is often called, the non-touching loop rule) to solve for the value of any node in this network. Before applying the rule, however, we must first define several additional terms.

A **first order loop** is defined as the product of the branches encountered in a journey starting from a node and moving in the direction of the arrows back to that original node. To illustrate this, let's start at node a_1. One first order loop is $S_{11}\Gamma_S$. Another first order loop is $S_{21}\Gamma_L S_{12}\Gamma_S$. If we now start at node a_2, we find a third first order loop, $S_{22}\Gamma_L$. Any of the other loops we encounter is one of these three first order loops.

A **second order loop** is defined as the product of any two non-touching first order loops. Of the three first order loops just found, only $S_{11}\Gamma_S$ and $S_{22}\Gamma_L$ do not touch in any way. The product of these two loops establishes the second order loop for this network. More complicated networks, involving feedback paths for example, might have several second order loops.

A **third order loop** is the product of any three non-touching first order loops. This example does not have any third order loops, but again, more complicated networks would have third order loops and even higher order loops.

Let's now suppose that we are interested in the value of b_l. In this example, b_S is the only independent variable because its value will determine the value of each of the other variables in the network. B_1, therefore, will be a function of b_S. To determine b_1, we first have to identify the paths leading from b_S to b_1. Following the arrows, we see two paths—(1) S_{11} and (2) $S_{21}\Gamma_L S_{12}$.

The next step is to find the non-touching loops with respect to the paths just found. Here, the path S_{11} and the first order loop, $S_{22}\Gamma_L$ have no nodes or branches in common. With this condition met, we can call $S_{22}\Gamma_L$ a non-touching loop with respect to the path S_{11}.

The other path, $S_{21}\Gamma_L S_{12}$, touches all of the network's first order loops, hence there are no non-touching loops with respect to this path. Again, in more complex networks, there would be higher order non-touching loops.

Let's now look at the non-touching loop rule itself (Fig. 48). This equation appears to be rather ominous at first glance, but once we go through it term by term, it will be less awesome. This rule determines the ratio of two variables, a dependent to an independent variable. (In our example, we are interested in the ratio b_1 to b_S.)

$$T = \frac{P_1[1 - \Sigma L(1)^{(1)} + \Sigma L(2)^{(1)} - \ldots] + P_2[1 - \Sigma L(1)^{(2)} \ldots}{1 - \Sigma L(1) + \Sigma L(2) - \Sigma L(3) + \ldots}$$

$$T = \frac{b_1}{b_s}$$

Figure 48

P_1, P_2, etc., are the various paths connecting these variables.

This term, $\Sigma L(1)^{(1)}$, is the sum of all first order loops that do not touch the first path between the variables.

This term, $\Sigma L(2)^{(1)}$, is the sum of all second order loops that do not touch that path, and so on down the line.

Now, this term, $\Sigma L(1)^{(2)}$, is the sum of all first order loops that do not touch the second path.

The denominator in this expression is a function of the network geometry. It is simply one minus the sum of all first order loops, plus the sum of all second order loops, minus the third order loops, and so on.

Let's now apply this non-touching loop rule to our network (Fig. 49). The ratio of b_1, the dependent variable, to b_S, the independent variable, is equal to the first path, S_{11}, multiplied by one minus the non-touching first order loop with respect to this path, $\Gamma_L S_{22}$.

$$\frac{b_1}{b_s} = \frac{S_{11}(1 - \Gamma_L S_{22}) + S_{21}\Gamma_L S_{12}(1)}{1 - (S_{11}\Gamma_s + S_{22}\Gamma_L + S_{21}\Gamma_L S_{12}\Gamma_s) + S_{11}\Gamma_s S_{22}\Gamma_L}$$

Figure 49

The second path, $S_{21}\Gamma_L S_{12}$, is simply multiplied by one because there are no non-touching loops with respect to this path.

The denominator is one minus the sum of first order loops plus the second order loop.

This concludes our example. With a little experience drawing flow graphs of complex networks, you can see how this technique will facilitate your network analysis. In fact, using the flow graph technique, we can now derive several expressions for power and gain that are of interest to the circuit designer.

First of all, we need to know the **power delivered to a load.** Remember that the square of the magnitudes of the incident and reflected waves has the dimension of power. The power delivered to a load is then the difference between the incident power and the reflected power, $P_{del} = |a|^2 - |b|^2$.

The **power available from a source** is **that** power delivered to a conjugately matched load. This implies that the reflection coefficient of the load is the conjugate of the source reflection coefficient, $\Gamma_S^* = \Gamma_L$.

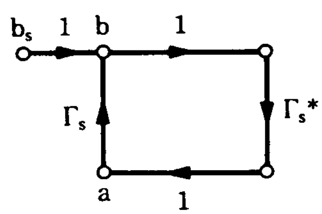


Figure 50

Looking at the flow graph describing these conditions (Fig. 50), we see that the power available from the source is:

$$P_{avs} = |b|^2 - |a|^2$$

Using the flow graph techniques previously described, we see that:

$$b = \frac{b_s}{1 - \Gamma_s \Gamma_s^*} \quad \text{and} \quad a = \frac{b_s \Gamma_s^*}{1 - \Gamma_s \Gamma_s^*}$$

The power available from the source reduces to (Fig. 51):

$$P_{avs} = \frac{|b_s|^2}{1 - |\Gamma_s|^2}$$

Figure 51

We can also develop voltage and power gain equations that will be useful in our amplifier designs using these flow graph techniques. For a two-port network, the voltage gain is equal to the total voltage at the output divided by the total voltage at the input,

$$Av = \frac{a_2 + b_2}{a_1 + b_1}$$

If we divide the numerator and denominator by bs, we can relate each of the dependent variables of the system to the one independent variable (Fig. 52a). Now we have four expressions or four ratios that we can determine from the non-touching loop rule.

a)

$$A_v = \frac{\frac{a_2}{b_s} + \frac{b_2}{b_s}}{\frac{a_1}{b_s} + \frac{b_1}{b_s}} = \frac{S_{21}\Gamma_L + S_{21}}{1(1 - S_{22}\Gamma_L) + S_{11}(1 - S_{22}\Gamma_L) + S_{21}\Gamma_L S_{12}}$$

b)

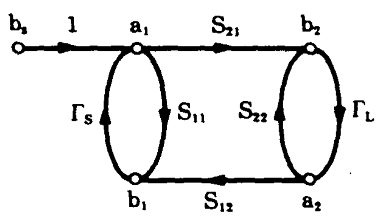


Figure 52

We can simplify this derivation by remembering that the denominator in the expression for the non-touching loop rule is a function of the network geometry. It will be the same for each of these ratios, and will cancel out in the end. So we only need to be concerned with the numerators of these ratios.

Let's trace through a couple of these expressions to firm up our understanding of the process (Fig. 52b). A_2 is connected to b_S through the path $S_{21}\Gamma_L$. All first order loops touch this path, so this path is simply multiplied by one. b_2 is connected to b_S through the path S_{21}. All first order loops also touch this path. a_1 is connected to b_S directly, and there is one non-touching loop, $S_{22}\Gamma_L$. We have already determined the ratio of b_1 to b_S, so we can simply write down the numerator of that expression. We have now derived the voltage gain of the two-port network.

The last expression we wish to develop is that for **transducer power gain**. This will be very important in the amplifier design examples contained in the final section of this seminar. Transducer power gain is defined as the power delivered to a load divided by the power available from a source.

$$G_t = \frac{P_{del}}{P_{avs}}$$

We have already derived these two expressions.

$$G_t = \frac{|b_2|^2 (1 - |\Gamma_L|^2)}{|b_s|^2 / (1 - |\Gamma_s|^2)}$$

What remains is to solve the ratio b_2 to b_S (Fig. 53a). The only path connecting b_S and b_2 is S_{21}. There are no non-touching loops with respect to this path. The denominator is the same as in the previous example: one minus the first order loops plus the second order loop. Taking the magnitude of this ratio, squaring, and substituting the result yields the expression for transducer power gain of a two-port network (Fig. 53b).

a)

$$\frac{b_2}{b_s} = \frac{S_{21}}{1 - S_{11}\Gamma_s - S_{22}\Gamma_L - S_{21}S_{12}\Gamma_L\Gamma_s + S_{11}\Gamma_s S_{22}\Gamma_L}$$

b)

$$G_T = \frac{|S_{21}|^2(1 - |\Gamma_s|^2)(1 - |\Gamma_L|^2)}{|(1 - S_{11}\Gamma_s)(1 - S_{22}\Gamma_L) - S_{21}S_{12}\Gamma_L\Gamma_s|^2}$$

Figure 53

Needless to say, this is not a simple relationship, as the terms are generally complex quantities. Calculator or computer routines will greatly facilitate the circuit designer's task.

Later, when designing amplifiers, we will see that we can simplify this expression by assuming that the amplifier is a unilateral device, or $S_{12} = 0$. In general, however, this assumption cannot be made, and we will be forced to deal with this expression.

One of the things you might want to do is to optimize or maximize the transducer gain of the network. Since the S-parameters at one frequency are constants depending on the device selected and the bias conditions, we have to turn our attention to the source and load reflection coefficients.

Stability Considerations

To maximize the transducer gain, we must conjugately match the input and the output. Before we do this, we will have to look at what might happen to the network in terms of stability—will the amplifier oscillate with certain values of impedance used in the matching process?

There are two traditional expressions used when speaking of stability: conditional and unconditional stability.

A network is **conditionally stable** if the real part of Z_{in} and Z_{out} is greater than zero for **some** positive real source and load impedances at a specific frequency.

A network is **unconditionally stable** if the real part of Z_{in} and Z_{out} is greater than zero for **all** positive real source and load impedances at a specific frequency.

It is important to note that these two conditions apply only at **one** specific frequency. The conditions we will now discuss will have to be investigated at many frequencies to ensure broadband stability. Going back to our Smith Chart discussion, we recall that positive real source and load impedances imply: $|\Gamma_S|$ and $|\Gamma_L| \leq 1$.

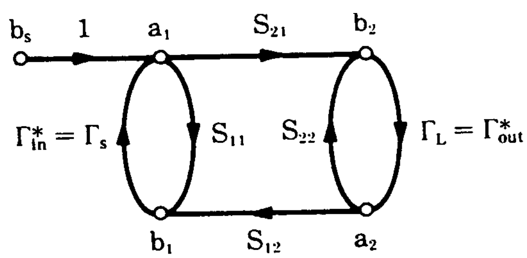


Figure 54

Let's look first at the condition where we want to conjugately match the network to the load and source to achieve maximum transducer gain (Fig. 54). Under these conditions, we can say that the network will be stable if this factor, K, is greater than one (Fig. 55). Conjugately matched conditions mean that the reflection coefficient of the source, Γ_S, is equal to the conjugate of the input reflection coefficient, Γ_{in}.

$\Gamma_S = \Gamma_{in}^*$

Γ_L is equal to the conjugate of the output reflection coefficient, Γ_{out}.

$\Gamma_L = \Gamma_{out}^*$

$$K = \frac{1 + |S_{11}S_{22} - S_{12}S_{21}|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}||S_{21}|} > 1$$

Figure 55

A **precaution** must be mentioned here. The K factor must not be considered alone. If we were operating under matched conditions in order to achieve maximum gain, we would have to consider the following: (1) What effect would temperature changes or drifting S-parameters of the transistor have on the stability of the amplifier? (2) We would also have to be concerned with the effect on stability as we substitute transistors into the circuit. (3) Would these changing conditions affect the conjugate match or the stability of the amplifier? Therefore, we really need to consider these other conditions in addition to the K factor.

Looking at the input and output reflection coefficient equations, we see a similarity of form (Fig. 56). The only difference is that S_{11} replaces S_{22}, and Γ_L replaces Γ_S.

$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{21}S_{12}\Gamma_s}{1 - S_{11}\Gamma_s}$$

Figure 56

If we set this equation, $|\Gamma_{in}|$, equal to one, a boundary would be established. On one side of the boundary, we would expect $|\Gamma_{in}| < 1$. On the other side, we would expect $|\Gamma_{in}| > 1$.

Let's first find the boundary by solving this expression (Fig. 57). We insert the real and imaginary values for the S-parameters and solve for Γ_L.

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \right| = 1$$

Figure 57

The solutions for Γ_L will lie on a circle. The radius of the circle will be given by this expression as a function of S-parameters (Fig. 58a).

a)

$$\text{radius} = r_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|$$

b)

$$\text{center} = C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}$$

$$\text{where: } \Delta = S_{11}S_{22} - S_{12}S_{21}$$

Figure 58

The center of the circle will have this form (Fig. 58b). Having measured the S-parameters of a two-port device at one frequency, we can calculate these quantities.

If we now plot these values on a Smith Chart, we can determine the locus of all values of Γ_L that make $|\Gamma_{in}| = 1$.

This circle then represents the boundary (Fig. 59). The area either inside or outside the circle will represent a stable operating condition.

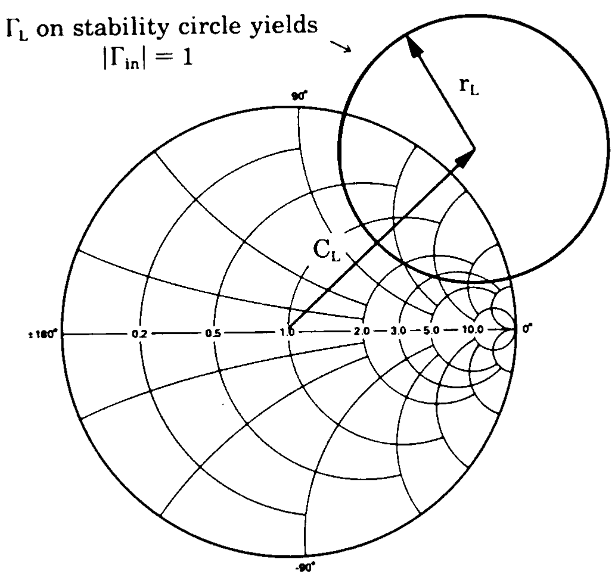


Figure 59

To determine which area represents this stable operating condition, let's make Z_L = 50 ohms, or $\Gamma_L = 0$. This represents the point at the center of the Smith Chart. Under these conditions, $|\Gamma_{in}| = |S_{11}|$.

Let's now assume that S_{11} has been measured and its magnitude is less than one. Γ_{in}'s magnitude is also less than one. This means that this point, $\Gamma_L = 0$, represents a stable operating condition. This region (Fig. 60) then represents the stable operating condition for the entire network.

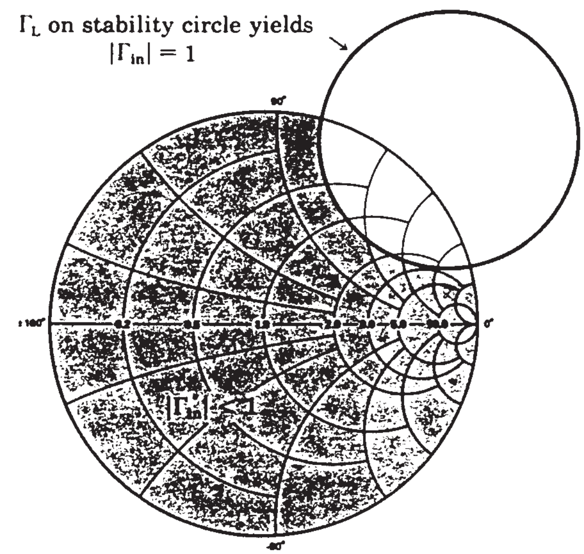


Figure 60

If we select another value of Γ_L that falls **inside** the stability circle, we would have an input reflection coefficient that would be greater than one, and the network would be potentially unstable.

If we only deal with passive loads, that is, loads having a reflection coefficient less than or equal to one, then we only have to stay away from those Γ_L's that are in this region (Fig. 61) to ensure stable operation for the amplifier we are designing. Chances are, we should also stay away from impedances in the border region, since the other factors like changing temperature, the aging of the transistors, or the replacement of transistors might cause the center or radius of the stability circle to shift. The impedance of the load could then fall in the expanded unstable region, and we would again be in trouble.

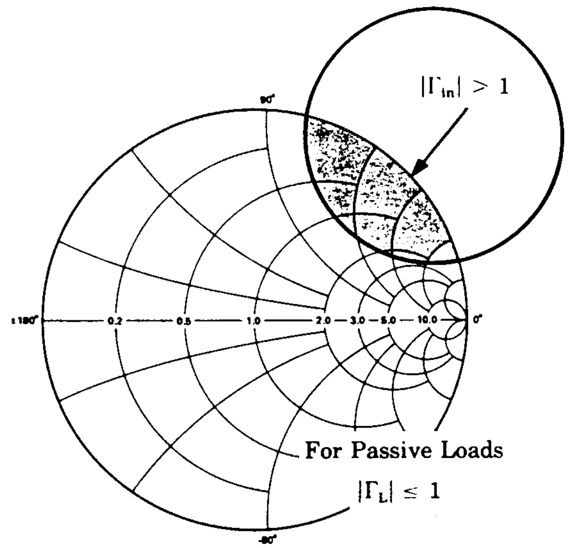


Figure 61

If, on the other hand, $|S_{11}| > 1$, with $Z_L = 50\ \Omega$, then this area would be the stable region and this region the unstable area (Fig. 62).

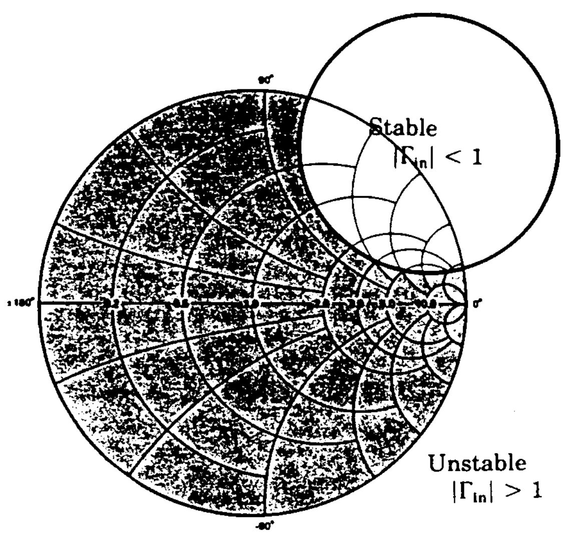


Figure 62

To ensure that we have an **unconditionally stable** condition at a given frequency in our amplifier design, we must be able to place any passive load on the network and drive it with any source impedance without moving into an unstable condition.

From a graphical point of view, we want to be sure that the stability circle falls completely outside the Smith Chart, and we want to make sure that the inside of the stability circle represents the unstable region (Fig. 63). The area outside the stability circle, including the Smith Chart, would then represent the stable operating region.

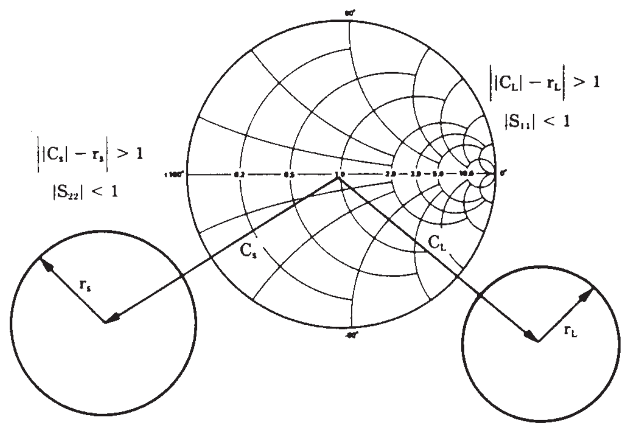


Figure 63

To satisfy this requirement, we must ensure that the magnitude of the vector, C_L, the distance from the center of the Smith Chart to the center of the stability circle, minus the radius of the stability circle, r_L, is greater than one. This means that the closest point on the stability circle would be outside the unit radius circle or Smith Chart.

To ensure that the region inside the Smith Chart represents the stable operating condition, the input or output impedance of the network must have a real part greater than zero when the network is terminated in 50 ohms. For completeness, we must also add the output stability circle to gain a better understanding of this concept. This means that the magnitude of S_{11} **and** S_{22} must be less than one.

One word of **caution** about stability.

S-parameters are typically measured at some particular frequency. The stability circles are drawn for **that** frequency. We can be sure that the amplifier will be stable at that frequency, but will it oscillate at some other frequency either inside or outside the frequency range of the amplifier?

Typically, we want to investigate stability over a broad range of frequencies and construct stability circles wherever we might suspect a problem. Shown here are the stability circles drawn for three different frequencies (Fig. 64). To ensure stability between f_1 and f_3, we stay away from impedances in this (shaded) area. While this process may sound tedious, we do have some notion based on experience where something may get us into trouble.

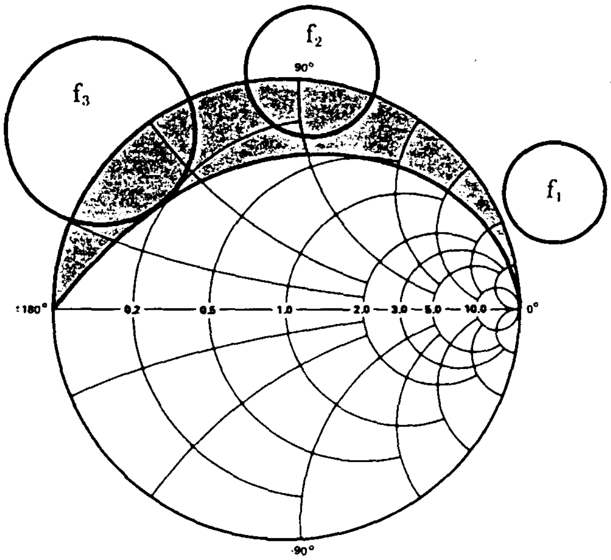


Figure 64

Stability is strongly dependent on the $|S_{12}|$ $|S_{21}|$ product (Fig. 65). $|S_{21}|$ is a generally decreasing function of frequency from f_β on.

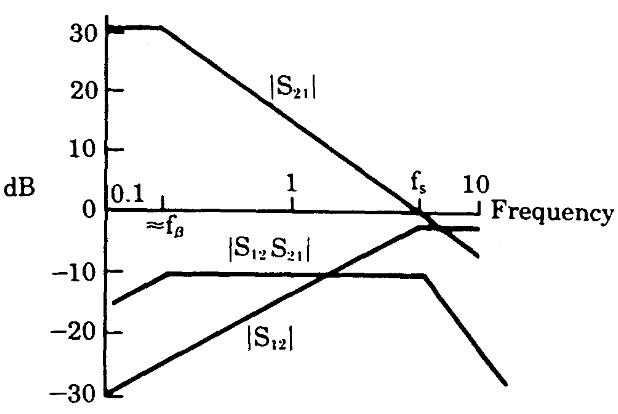


Figure 65

$|S_{12}|$ is an increasing function.

Looking at the $|S_{12}|$ $|S_{21}|$ product, we see that it increases below f_β, flattens out, then decreases at higher frequencies.

It is in this flat region that we must worry about instability.

On the other hand, if we synthesize elements such as inductors by using high impedance transmission lines, we might have capacitance rather than inductance at higher frequencies, as seen here on the Impedance Phase plot (Fig. 66). If we suspect that this might cause oscillation, we would investigate stability in the region where the inductor is capacitive. Using tunnel diodes having negative impedance all the way down to dc, we would have to investigate stability right on down in frequency to make sure that oscillations did not occur outside the band in which we are working.

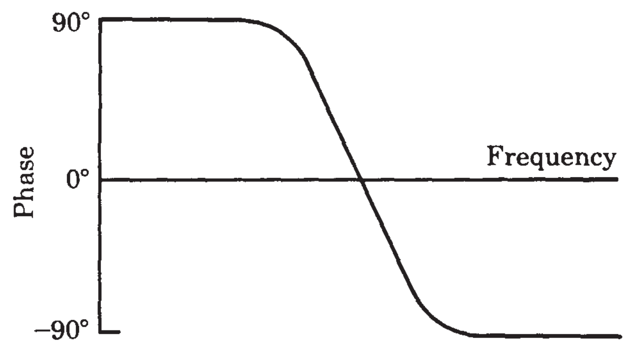


Figure 66